# Research Report

## Evaluation of a Newly Developed Observation Operators for Assimilating Radar Radial Velocity Observations in GSI to Improve Storm Forecast Using the HRRR System

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### 1. Background

The IVAP (Integrating Velocity-Azimuth Process) (Liang 2007) based radar radial velocity assimilation operator was introduced into the GSI system by Chen and Liang (2017). To evaluate this operator and merge it to the master branch of unified GSI repsoitroy, DTC granted a visitor program "Evaluation of a Newly Developed Observation Operators for Assimilating Radar Radial Velocity Observations in GSI to Improve Storm Forecast Using the HRRR System".

Dr. Xudong Liang visited DTC during 15 Feb. to 16 Mar. 2019. As part of this visit, the codes for the operator were merged with the GSI master branch and a case was tested with the help of Dr. Guoqing Ge. More analyses were done after the visit.

### 2. Introduction of the operator

The traditional observation operator for Doppler radar radial velocity is

$$V_r = u\sin\theta\cos\phi + v\cos\theta\cos\phi + w\sin\phi, \qquad (1)$$

where $(u,v,w)$ are wind components in the Cartesian coordinate of (x, y, z). According to Liang (2007) and Luo (2014), there are two spatial distribution characteristics of radial wind, which can be obtained by multiplying $\sin\theta$ or $\cos\theta$ on both sides of Eq. (1) and summing within a given area $\Omega$ :

$$\begin{cases}\sum_{\Omega} V_r \sin\theta = \sum_{\Omega} u \sin^2\theta\cos\varphi + \sum_{\Omega} v \sin\theta\cos\theta\cos\varphi + \sum_{\Omega} w \sin\theta\sin\varphi \\ \sum_{\Omega} V_r \cos\theta = \sum_{\Omega} u \sin\theta\cos\theta\cos\varphi + \sum_{\Omega} v \cos^2\theta\cos\varphi + \sum_{\Omega} w \cos\theta\sin\varphi \end{cases}. \quad (2)$$

If we define the averaged wind within the area $\Omega$ using $\left(\bar{u},\bar{v},\bar{w}\right)$, Eq. (2) can be expressed as follows,

$$\begin{cases}\sum_{\Omega} V_r \sin\theta = \bar{u}\sum_{\Omega} \sin^2\theta\cos\varphi + \bar{v}\sum_{\Omega} \sin\theta\cos\theta\cos\varphi + \bar{w}\sum_{\Omega} \sin\theta\sin\varphi \\ \sum_{\Omega} V_r \cos\theta = \bar{u}\sum_{\Omega} \sin\theta\cos\theta\cos\varphi + \bar{v}\sum_{\Omega} \cos^2\theta\cos\varphi + \bar{w}\sum_{\Omega} \cos\theta\sin\varphi \end{cases}. \quad (3)$$

Dividing both sides of Eq. (3) by $\sum_{\Omega}\sin^2\theta\cos\varphi$ or $\sum_{\Omega}\cos^2\theta\cos\varphi$, we have

$$\begin{cases}\dfrac{\sum_{\Omega} V_r \sin\theta}{\sum_{\Omega}\sin^2\theta\cos\varphi} = \bar{u} + \bar{v}\dfrac{\sum_{\Omega}\sin\theta\cos\theta\cos\varphi}{\sum_{\Omega}\sin^2\theta\cos\varphi} + \bar{w}\dfrac{\sum_{\Omega}\sin\theta\sin\varphi}{\sum_{\Omega}\sin^2\theta\cos\varphi} \\ \dfrac{\sum_{\Omega} V_r \cos\theta}{\sum_{\Omega}\cos^2\theta\cos\varphi} = \bar{u}\dfrac{\sum_{\Omega}\sin\theta\cos\theta\cos\varphi}{\sum_{\Omega}\cos^2\theta\cos\varphi} + \bar{v} + \bar{w}\dfrac{\sum_{\Omega}\cos\theta\sin\varphi}{\sum_{\Omega}\cos^2\theta\cos\varphi} \end{cases}. \quad (4)$$

As shown in Liang (2007), Eq. (4) can be used to retrieve all components of the mean wind within the given area $\Omega$. The left and right sides of Eq. (4) are defined in observation and analysis spaces, respectively, as follows,

$$\begin{cases} Y_1 = \dfrac{\sum_{\Omega} V_r \sin\theta}{\sum_{\Omega}\sin^2\theta\cos\varphi} \\ Y_2 = \dfrac{\sum_{\Omega} V_r \cos\theta}{\sum_{\Omega}\cos^2\theta\cos\varphi} \end{cases} \quad (5)$$

and

$$\begin{cases} H_1 = \bar{u} + \bar{v}\dfrac{\sum\limits_{\Omega}\sin\theta\cos\theta\cos\varphi}{\sum\limits_{\Omega}\sin^2\theta\cos\varphi} + \bar{w}\dfrac{\sum\limits_{\Omega}\sin\theta\sin\varphi}{\sum\limits_{\Omega}\sin^2\theta\cos\varphi} \\ H_2 = \bar{u}\dfrac{\sum\limits_{\Omega}\sin\theta\cos\theta\cos\varphi}{\sum\limits_{\Omega}\cos^2\theta\cos\varphi} + \bar{v} + \bar{w}\dfrac{\sum\limits_{\Omega}\cos\theta\sin\varphi}{\sum\limits_{\Omega}\cos^2\theta\cos\varphi} \end{cases} . \qquad (6)$$

Eq. (6) can be used as an observation operator instead of Eq. (1) in radar radial wind assimilation.

Equations (5) and (6) can be introduced into the 3DVAR cost function

$$J = J_b + J_o, \qquad (7)$$

where $J_b$ and $J_o$ are background and observation terms, respectively. The radar wind part $J_o^r$ in the observation term $J_o$ is

$$J_o^r = \frac{1}{2}\left\{\begin{bmatrix} H_1 - Y_1 \\ H_2 - Y_2 \end{bmatrix}^T R^{-1} \begin{bmatrix} H_1 - Y_1 \\ H_2 - Y_2 \end{bmatrix}\right\}, \qquad (8)$$

where $Y_1$ and $Y_2$ are the observations (in observation space) defined by Eq. (5); $H_1$ and $H_2$ are the new observation operators (in analysis space) defined by Eq. (6). $B$ and $R$ are the background error covariance and observation error variance matrices, respectively, and

$$\begin{bmatrix} H_1 - Y_1 \\ H_2 - Y_2 \end{bmatrix} = \begin{bmatrix} (\dfrac{1}{N}\sum\limits_{\Omega} u) + (\dfrac{1}{N}\sum\limits_{\Omega} v)\dfrac{\sum\limits_{\Omega}\sin\theta\cos\theta\cos\varphi}{\sum\limits_{\Omega}\sin^2\theta\cos\varphi} - \dfrac{\sum\limits_{\Omega} V_r\sin\theta}{\sum\limits_{\Omega}\sin^2\theta\cos\varphi} \\ (\dfrac{1}{N}\sum\limits_{\Omega} u)\dfrac{\sum\limits_{\Omega}\sin\theta\cos\theta\cos\varphi}{\sum\limits_{\Omega}\cos^2\theta\cos\varphi} + (\dfrac{1}{N}\sum\limits_{\Omega} v) - \dfrac{\sum\limits_{\Omega} V_r\cos\theta}{\sum\limits_{\Omega}\cos^2\theta\cos\varphi} \end{bmatrix}. \qquad (9)$$

## 3. Implement of the new operator in GSI

Generally, the improved observation operator can be applied in GSI in three steps. First, the observations of $Y_1$ and $Y_2$ are calculated using Eq. (5) based on the radial velocity $V_r$ in the radar coordinate system; and two coefficients $\frac{\sum_{\Omega}\sin\theta\cos\theta\cos\varphi}{\sum_{\Omega}\cos^2\theta\cos\varphi}$ and $\frac{\sum_{\Omega}\sin\theta\cos\theta\cos\varphi}{\sum_{\Omega}\sin^2\theta\cos\varphi}$ are recorded. Second, the area-averaged wind components of $\bar{u}$ and $\bar{v}$ are obtained from the model wind field. Third, the values of $H_1$ and $H_2$ are computed based on Eq. (6).

If using these three steps, a pre-processing procedure should be done using a given size and shape of domain $\Omega$ to calculate $Y_1$, $Y_2$, $\frac{\sum_{\Omega}\sin\theta\cos\theta\cos\varphi}{\sum_{\Omega}\cos^2\theta\cos\varphi}$, and $\frac{\sum_{\Omega}\sin\theta\cos\theta\cos\varphi}{\sum_{\Omega}\sin^2\theta\cos\varphi}$ . On the other hand, it should be re-calculated if the size or shape of domain $\Omega$ is changed. This recalculation necessity is inconvenient.

If we define the average value $(\bar{u}, \bar{v})$ in model, and define the observation operator as (to simplify, the vertical motion *w* is omitted):

$$V_r = \bar{u}\sin\theta\cos\varphi + \bar{v}\cos\theta\cos\varphi \quad (10),$$

which is similar to Eq. (1). When we have some observations within a given area with the average winds $(\bar{u}, \bar{v})$, we get,

$$\begin{cases} V_{r1} = \bar{u}\sin\theta_1\cos\varphi_1 + \bar{v}\cos\theta_1\cos\varphi_1 \\ V_{r2} = \bar{u}\sin\theta_2\cos\varphi_2 + \bar{v}\cos\theta_2\cos\varphi_2 \\ V_{r3} = \bar{u}\sin\theta_3\cos\varphi_3 + \bar{v}\cos\theta_3\cos\varphi_3 \\ \ldots\ldots \end{cases} \quad (11).$$

Define

$$f=\sum_{i}^{N}[V_{ri}-(\bar{u}\sin\theta_i\cos\varphi_i+\bar{v}\cos\theta_i\cos\varphi_i)]^2 \quad (12).$$

To get the minimum of $f$, according to

$$\frac{\partial f}{\partial \bar{u}}=-\sum_{i}^{N}2[V_{ri}-(\bar{u}\sin\theta_i\cos\varphi_i+\bar{v}\cos\theta_i\cos\varphi_i)]\sin\theta_i\cos\varphi_i \quad (13\text{-}1)$$

and

$$\frac{\partial f}{\partial \bar{v}}=-\sum_{i}^{N}2[V_{ri}-(\bar{u}\sin\theta_i\cos\varphi_i+\bar{v}\cos\theta_i\cos\varphi_i)]\cos\theta_i\cos\varphi_i \quad (13\text{-}2),$$

given $\frac{\partial f}{\partial \bar{u}}=0$ and $\frac{\partial f}{\partial \bar{v}}=0$ get

$$\begin{cases}\sum_{i}^{N}2V_{ri}\sin\theta_i\cos\varphi_i=\sum_{i=1}^{N}(\bar{u}\sin\theta_i\cos\varphi_i+\bar{v}\cos\theta_i\cos\varphi_i)\sin\theta_i\cos\varphi_i\\ \sum_{i}^{N}2V_{ri}\cos\theta_i\cos\varphi_i=\sum_{i=1}^{N}(\bar{u}\sin\theta_i\cos\varphi_i+\bar{v}\cos\theta_i\cos\varphi_i)\cos\theta_i\cos\varphi_i\end{cases} \quad (14).$$

Equation (14) has the same form as Eq. (2). It means that the optimal solutions of Eq. (11) are same as those of Eq. (2). Thereafter, another form of the operator is defined according to Eq. (11). In this form, the averaged wind speed components $(\bar{u},\bar{v})$ are calculated during the assimilation processes. To calculate the averaged wind speed, the Gaussian Smoothing Filter is introduced instead of bi-linear interpolation in GSI when assimilating radar radial velocity. In the Gaussian Smoothing Filter, the average wind speed is calculated using

$$\bar{u}=\frac{1}{W}\sum_{i=1}^{N}u_i w_i \quad (15),$$

where $u_i$ is the wind speed at model grid point $i$ with distance $x, y$ to the observation point (location of radar observation) as shown in figure 1., $w_i$ is the weighting calculated using

$$w = \frac{1}{2\pi\sigma^2}\exp(\frac{x^2 + y^2}{2\sigma^2}) \quad (16),$$

where $\sigma$ is given influence radius. $W$ is the sum of $w_i$.

## 4. The codes of the operator

The Gaussian Smoothing Filter is introduced in intsrw.f90. The variable for weightings $w_i$ of surrounding 32 points with index of j1_32 is defined as

```
integer(i_kind),dimension(32):: j1_32    !index of the surrounding points
real(r_kind),dimension(32) :: w1_32 ! weightings
```

The weightings are calculated using w32.h according to equation (16).

```
ss=1.0
dx =w3/(w1+w3)
dx1=w1/(w1+w3)
dy =w2/(w1+w2)
dy1=w1/(w1+w2)
ds =w5/(w1+w5)
ds1=w1/(w1+w5)
w1_32(1 )=ss**(dx **2 +dy**2)
w1_32(2 )=ss**(dx **2+dy1**2)
w1_32(3 )=ss**(dx1**2+dy **2)
w1_32(4 )=ss**(dx1**2+dy1**2)
w1_32(5 )=w1_32(1)
w1_32(6 )=w1_32(2)
w1_32(7 )=w1_32(3)
w1_32(8 )=w1_32(4)
w1_32(9 )=ss**((dx +1)**2+(dy +1)**2)
w1_32(10)=ss**((dx +1)**2+ dy       **2)
w1_32(11)=ss**((dx +1)**2+ dy1      **2)
w1_32(12)=ss**((dx +1)**2+(dy1+1)**2)
w1_32(13)=ss** (dx       **2+(dy +1)**2)
w1_32(14)=ss** (dx       **2+(dy1+1)**2)
w1_32(15)=ss** (dx1      **2+ dy1      **2)
w1_32(16)=ss** (dx1      **2+(dy1+1)**2)
w1_32(17)=ss**((dx1+1)**2+(dy +1)**2)
w1_32(18)=ss**((dx1+1)**2+ dy       **2)
w1_32(19)=ss**((dx1+1)**2+ dy1      **2)
w1_32(20)=ss**((dx1+1)**2+(dy1+1)**2)
```

```
do p_loop=21,32
  w1_32(p_loop)=w1_32(p_loop-12)
enddo
ss=0.
do p_loop=1,32
  w1_32(p_loop)=1./sqrt(w1_32(p_loop))
  ss=ss+w1_32(p_loop)
enddo
do p_loop=1,32
  w1_32(p_loop)=w1_32(p_loop)/ss
enddo
```

*The operator is*

```
do p_loop=1,32
   valu=valu+w1_32(p_loop)*su(j1_32(p_loop))
   valv=valv+w1_32(p_loop)*sv(j1_32(p_loop))
enddo
val=valu*rwptr%cosazm_costilt+valv*rwptr%sinazm_costilt
```

where *valu* and *valv* are the averaged winds.

The codes for the adjoint of the operator are in stprw.f90 correspondingly.

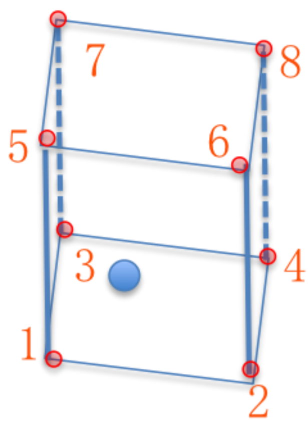



Figure 1. The observation point (blue point) and the surrounding model grid points (red point).

## 5. Experiments

A case is tested using the new operator. The initial time is 00Z, June 27, 2018 with the model domain shown in figure 2.

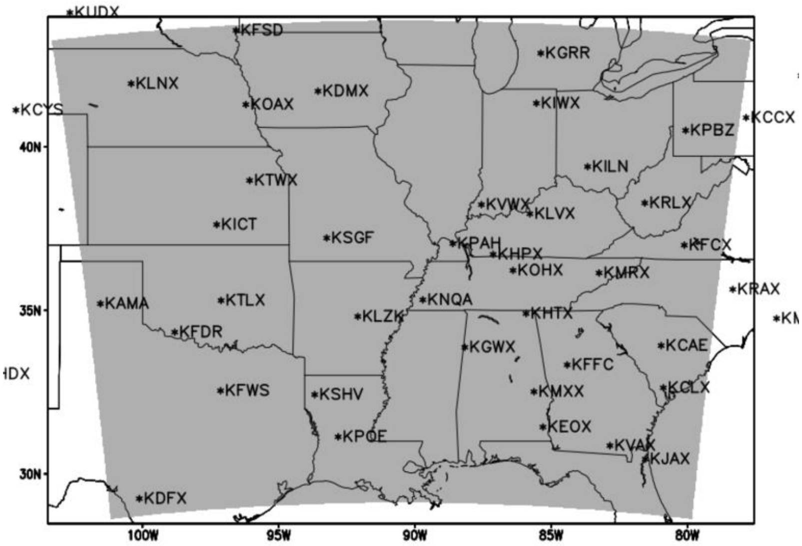



Figure 2 Model domain.

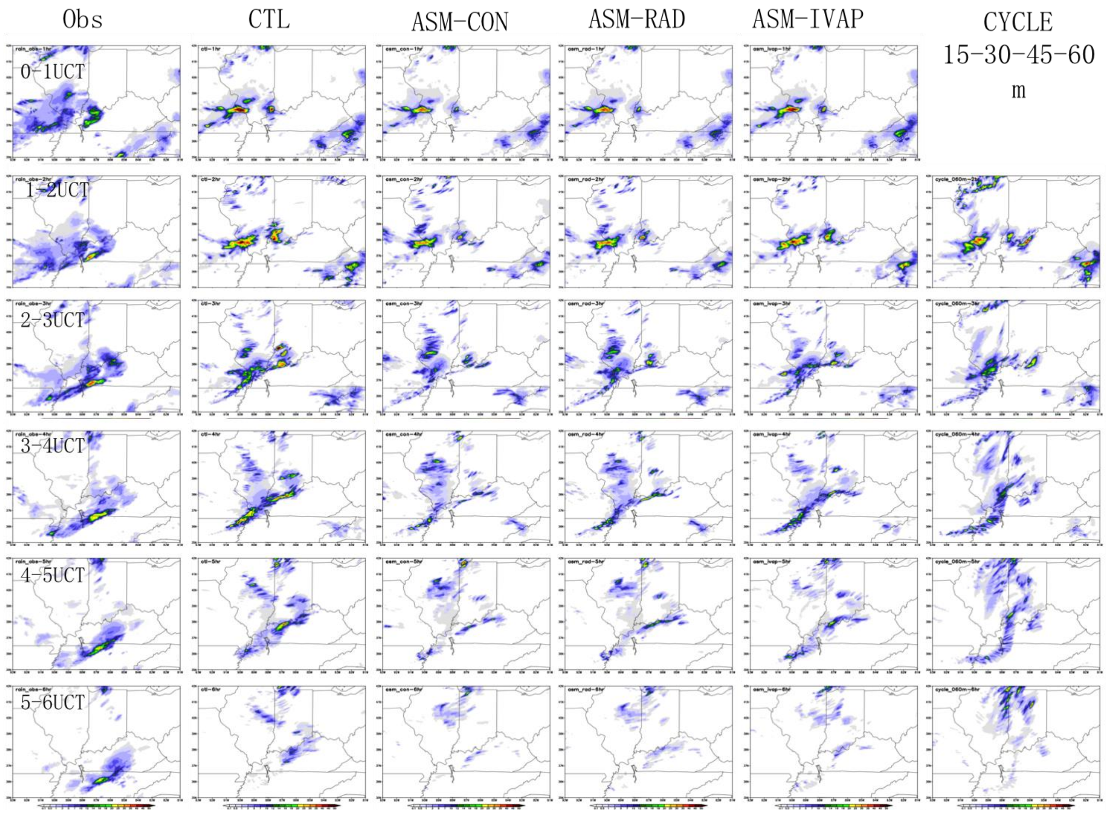
Five experiments were carried out. In the control run, no observations were assimilated. Radar data besides the conventional data were assimilated using the traditional operator in *asm_rad* and using the new operator in the *asm_ivap* experiment. In the *asm_con* experiment, only the conventional data were assimilated. In the *cycle* experiment, the radar data were assimilated in 5 cycles with an interval of 15 minutes. The results were verified according to the Stage IV Data 1-hour precipitation data.

Table 1 The five experiments.

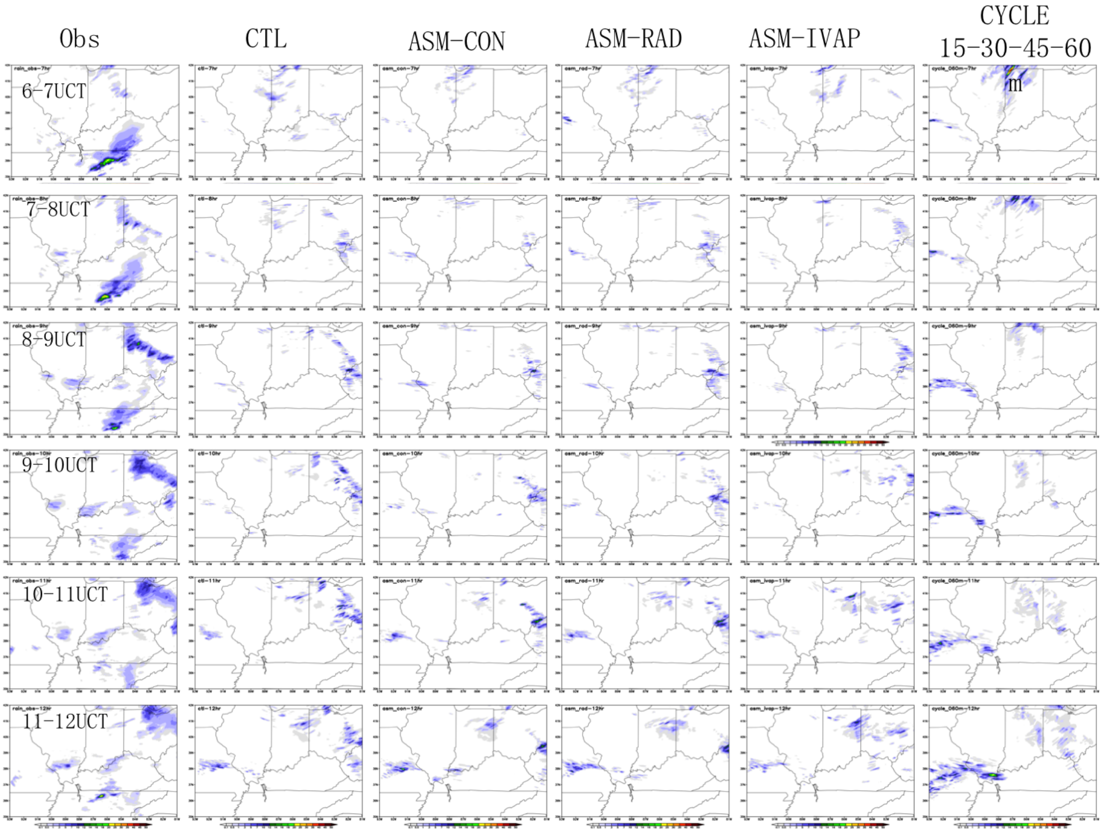
| Experiment | Data and method |
|---|---|
| Ctl | Without data assimilation |
| asm_con | Data assimilation without radar data |
| asm_rad | Data assimilation with radar data using traditional operator |
| asm_ivap | Data assimilation with radar data using IVAP based operator |

| | |
|---|---|
| cycle | Data assimilation with radar data using IVAP based operator in cycle mode |

As shown in figure 3, the precipitation forecast of the 5 experiments indicate that assimilating of the radar radial velocity are helpful for improving the forecast skill.



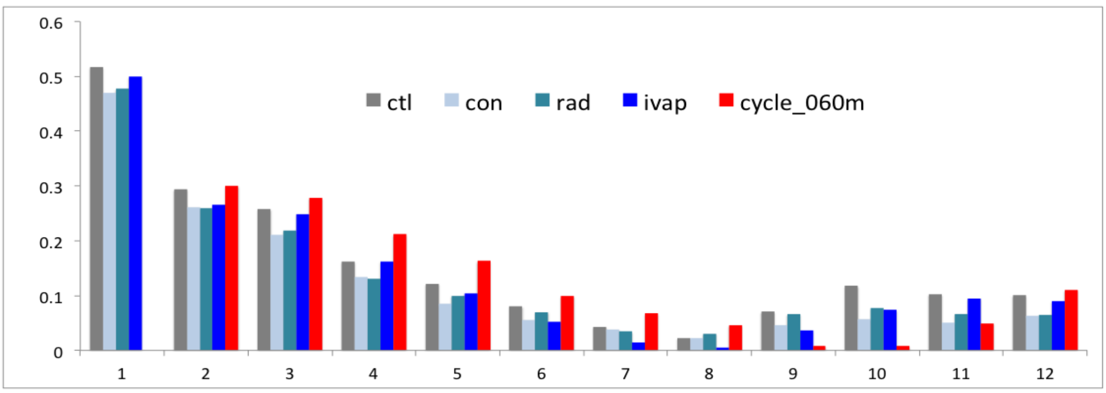

(a)



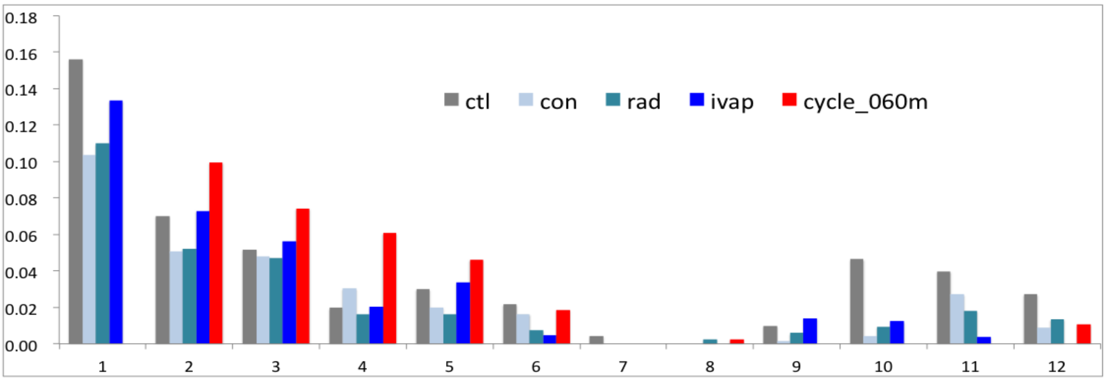

(b)

Figure 3 Observation and forecast of 1 hour accumulated precipitation from (a) 1-5 and (b) 6-11 hour.

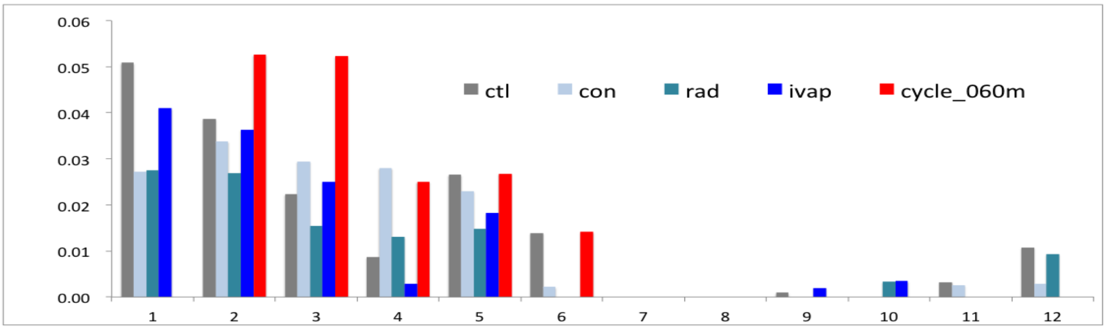
The TS score also shows that radar radial velocity is useful, especially using the new operator.




(a) 0.1 mm/hr




(b) 2.5 mm/hr




(c) 5 mm/hr

Figure 4 TS score with threshold of (a) 0.1 mm/hr, (b) 2.5 mm/hr, and (c) 5 mm/hr.

## 6. Conclusions and suggestions.

- A new forward operator for radar radial velocity assimilation was proposed based on the IVAP method, which can use more information of radar observation.
- The operator was added in the GSI codes.
- Primary results of the experiments have shown positive impacts.
- More works on tuning and testing should be done in the next step, not only for the operator but also for the data pre-processing (such as super-ob, thinning).

## References

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