# Eliminating grid imprinting with a Conformal Cubic Overset Grid

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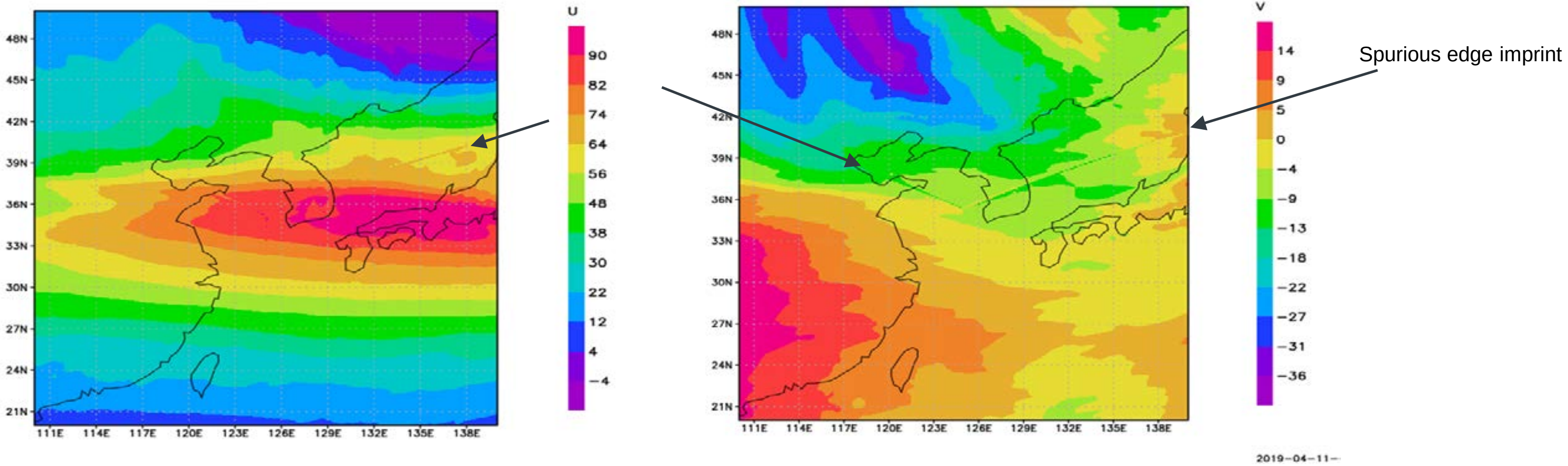
# The Problem

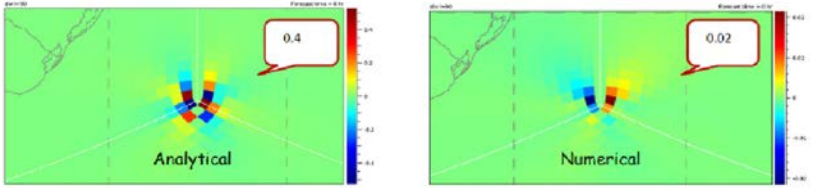
Cubic grids based on a separate gnomonic projection for each face of the cube possess a sharp angular discontinuity across each edge and corner.
This can lead to unsightly spurious imprints in fields derived from forecasts:

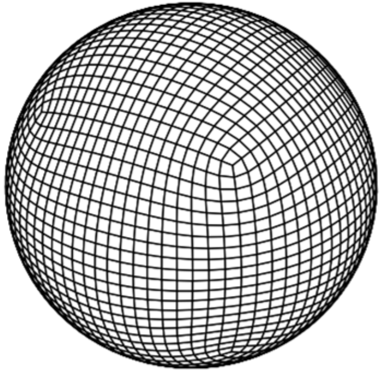



(Thanks to Fanglin Yang and Yali Ma for providing these figures.)

The imprinting does not entirely go away even when we choose a cubic grid geometry that remains smooth and continuous across each edge. For example, the “UJ cubed sphere”, developed at EMC as a proposed framework for extending their NMMB model to a global system, was found to suffer grid imprinting at the vertices, where the curvature of grid lines becomes singular.

Spurious divergence at the corner of the UJ cube in idealized shallow water nondivergent flow computed using two styles of the treatment of the metric terms near the singularity; in neither case is the problem solved.

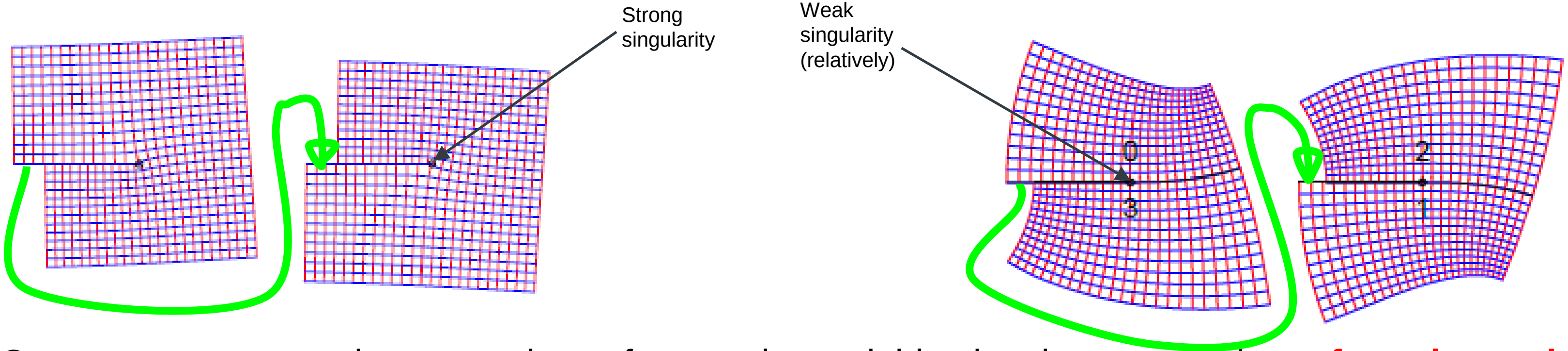


The topological reality is that no **SINGLE-VALUED** mapping to a griddable planar region can avoid the necessary presence of singularities.
But, if we remove the restriction that the mapping be single-valued, we can look for partially self-overlapping grids that are not only smooth, but also **CONFORMAL**.

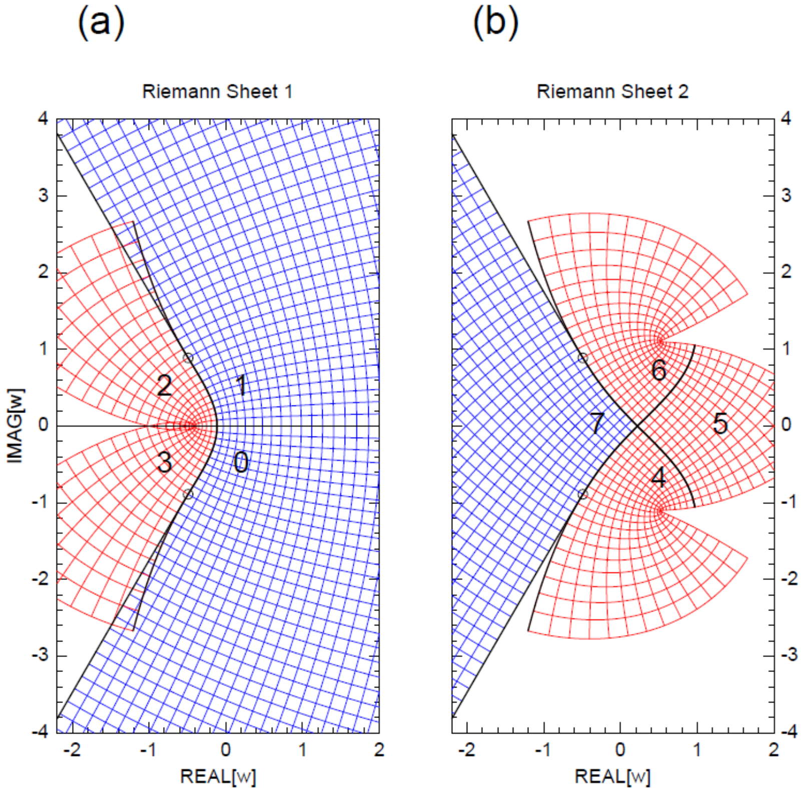
# Conformal mapping and Riemann surfaces

$w = z + \pm 0.5i\sqrt{z},$ $w = z + \pm 0.003iz^2\sqrt{z},$




Convergent power series expansions of a complex variable give rise to smooth **conformal mappings.**
When the expansion is in half-integer powers, the conformal mapping can result in a solution best described on a two-sheeted **Riemann surface.**
There is a singular **branch point** at the center, but when the first half-odd-integer power is of a relatively high degree (right panels), this singularity is a weak one.
(The grids here are formed by constant real, and constant imaginary, values.)

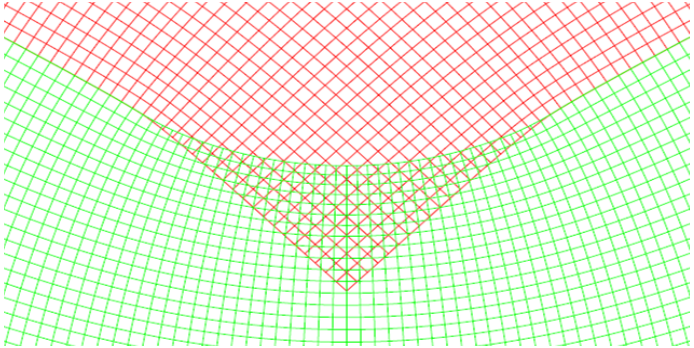
# More exotic Riemann surfaces



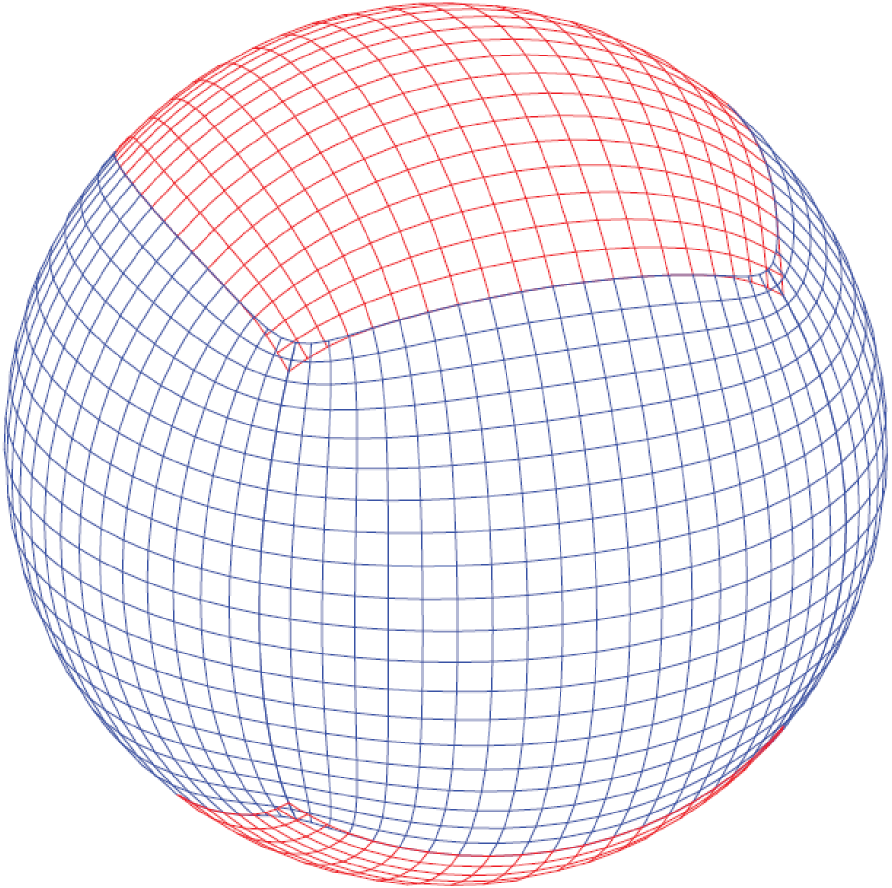

Under the right conditions, it is possible to combine power expansions about several distinct expansion centers into a single coherent analytic function.

In the figures to the left, the half-integer power expansions about the two branch points (small circles) are combined with two suitably constructed fractional-power expansions about the two 'points at infinity' (on the two sheets) such that, outside of the branch points, three blue quadrants of the grid wrap around together, while five red quadrants (containing additional mapping singularities) wrap around the corner region. Although this solution is planar, we can think of the blue portion of the grid as a model for the idealized grid around a cube's corner.

The method by which the power series are brought into mutual agreement (a variant of a ‘Schwarz’ iteration) is essentially the same type of method by which copies of this planar solution can themselves be combined to form the complete cubic grid. The blue portions of the preceding grid become the physical grid panels, which now overlap near each corner region of the cube.



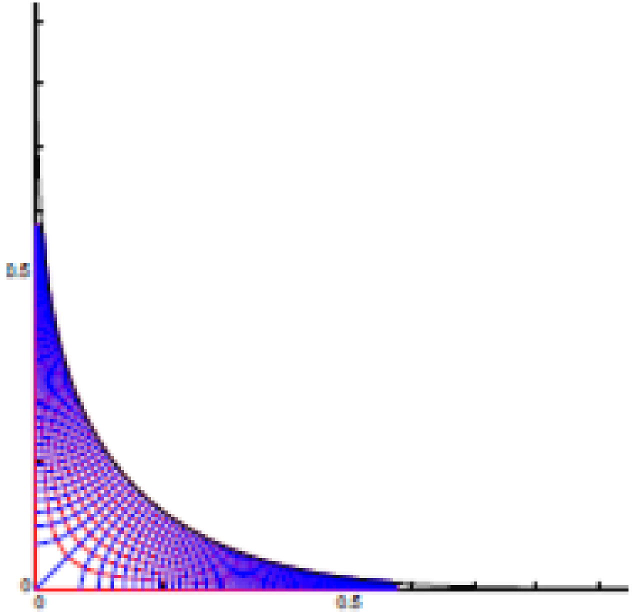
Detail of a ‘bicorn’ region of the overset.



# Remarks

The conformal cubic overset grids that we can generate by this procedure have their geometry completely specified by:

1) Their branch point order of smoothness
2) The position of the branch points pairs.



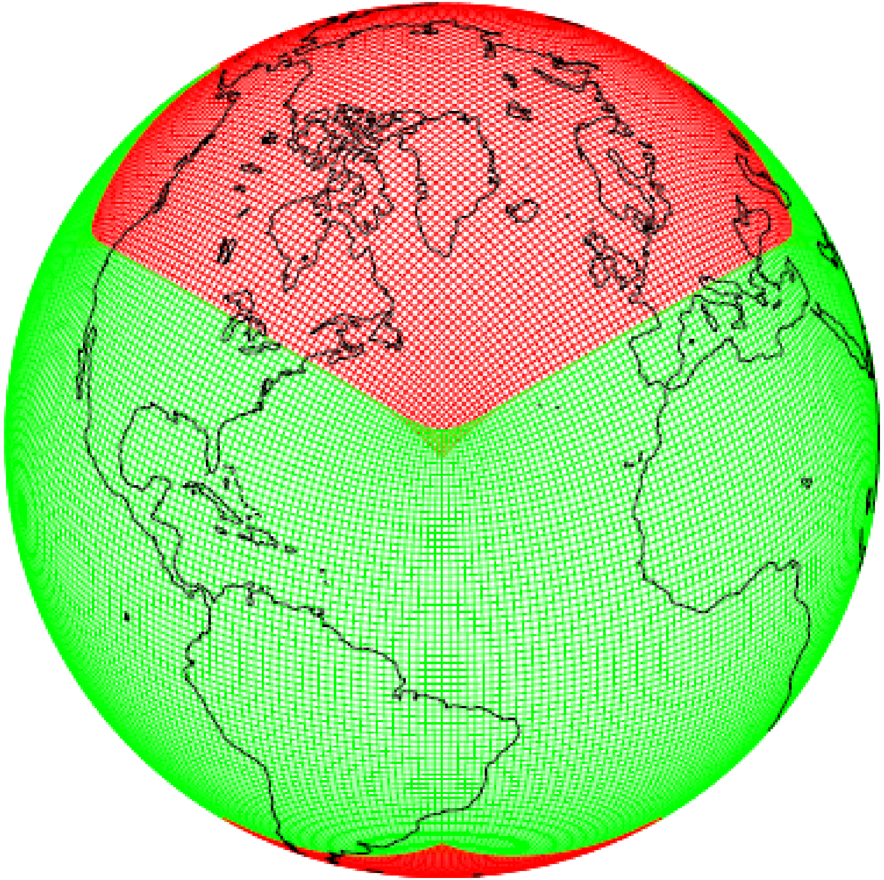

A planar conformal image (e.g., via stereographic projection) of the bicorn overset can itself be conformally mapped to an infinite strip (see figure at right) and one family of iso-lines, or potentials, that mediate between the curved boundary and the pair of straight boundaries, can be used to define the contours of equal blending weight when solutions on the overset are smoothly blended.

The numerics of the FV3 model will not ‘feel’ the singularities at the branch points because the mapping is sufficiently smooth there.

The solution reconciliation (interpolation and blending) performed each time step need only be done inside the tiny oversets, so the additional numerical burden is negligible.

The grid resolution, although not uniform, does not vary greatly (unlike the pure conformal cubic grid).

The conformal grid provides potential savings in the model itself, as the covariant and contravariant vector (and tensor) representations essentially coincide and a number of ‘cross terms’ become redundant in the dynamics.



# Conservation and monotonicity

The small size of the overlapping zones will allow for an efficient preservation of mass conservation and monotonicity of the solution.

Mass conservation can be achieved by monitoring the budget of incoming and outcoming fluxes in the overlapping zone and making corrections so that blending of the solutions from different surfaces may not produce new maxima or minima nor generate or reduce mass, that is, by a posteriori constraint-restoration method.

We  are in the process of formulating a conformal cubic overset grid version of the FV3 model using the methods we have described, and should soon be in a position to decide whether these methods eliminate grid imprinting, as we hope.