The Extended Schmidt Gnomonic grid for Regional Applications

NATIONAL WEATHER SERVICE

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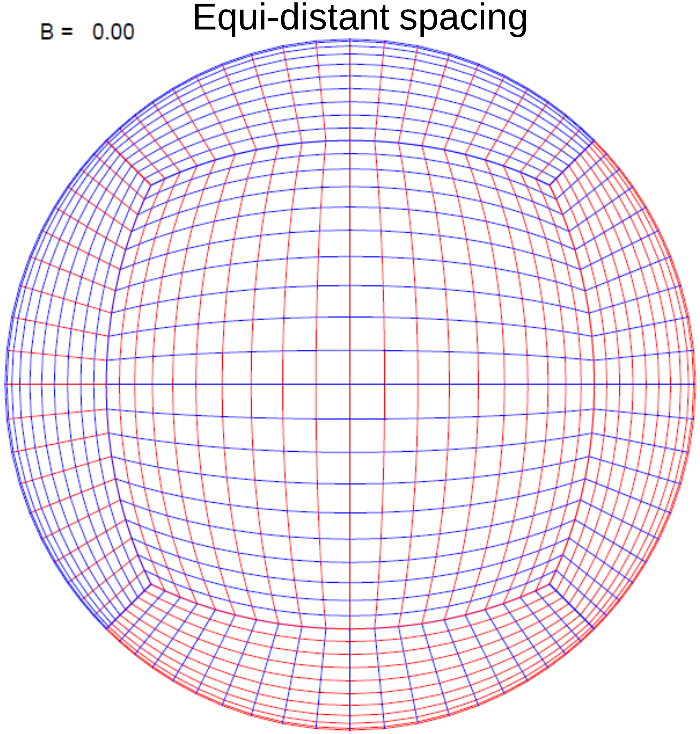
Context

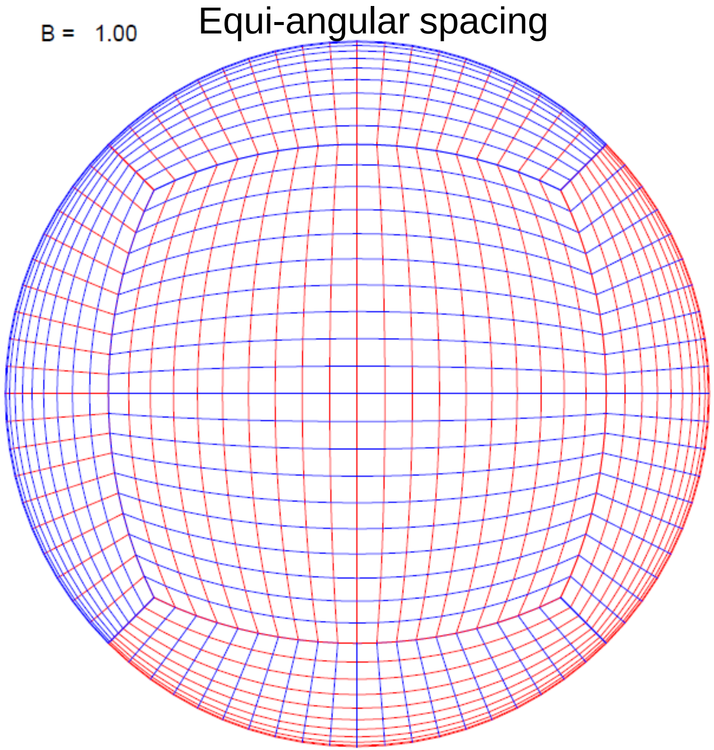
The FV3 global model uses a cubic geometry

On each face of the cube the grid is of the gnomonic kind (central projection to a plane) --the lines are great circles.

One parameter is available to control the profile of spacing between the lines in each of the two families forming the grid.

'Equi-distant' and 'equi-angular' grids belong to the same family (although this may not seem obvious).



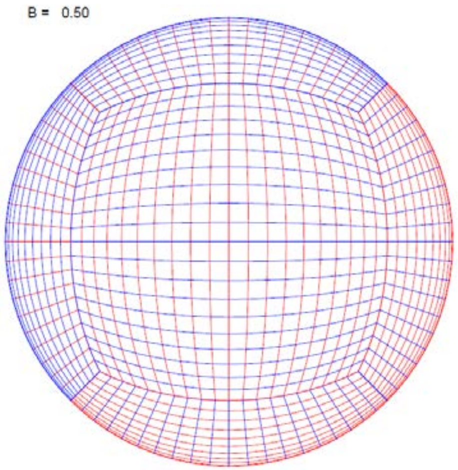


On the left, the lines of the grid would appear equally-spaced projected onto the tangent plane, but are more tightly packed at the edge of the face on the sphere. On the right, the angular spacing is uniform.

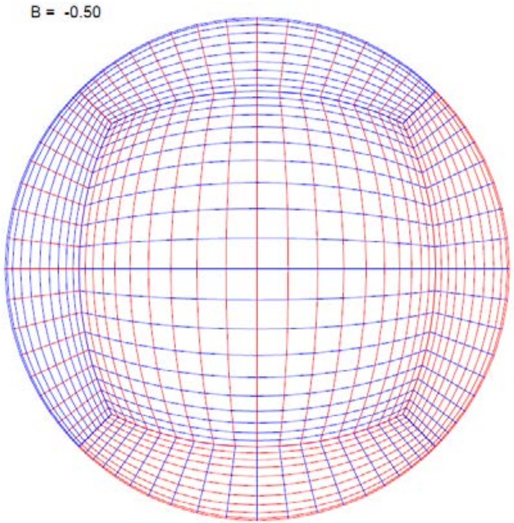
The meaning of parameter, **B**:

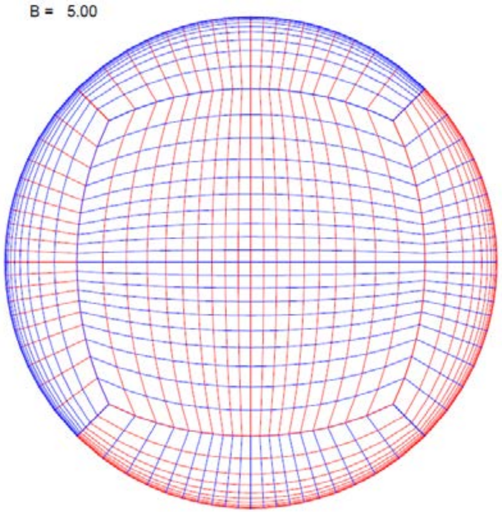
On an arc-line, angular distance arccos(sqrt{**B**}) from the face center, the transverse grid lines which intersect it do so at a uniform spacing on the sphere (the **B**=0 case being obtained as a limit).

With this purely geometrical definition of the spacing parameter, B, in the range [0,1] we can find the FV3-GFS grid used operationally, where the spacing is uniform along an edge of the cube, to be the case, B=0.5:



But ALGEBRAICALLY, there is no obstacle to analytically-continuing parameter B into the extended additional ranges, B<0 and B>1 as well!

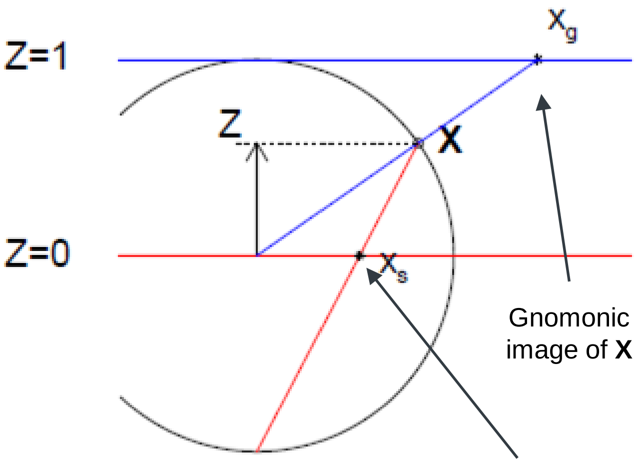




<- Although these extreme grids are not viable choices for a global grid, they do show that a greater parameter range is available when we construct a limited area grid.

The FV3 cubed-sphere grid is equipped with a second parameter -- the Schmidt conformal refinement factor, S. It works as follows:

The gnomonic grid, formed by families of straight lines on the tangent plane (blue, in the figure), and centrally back-projected to a unit sphere (point **X**), are projected STEREOGRAPHICALLY (from the south pole in this figure) to the equatorial plane (red). Now, if we **scale** this stereographic plane by the factor, 1/S, and back-project (stereographically) onto the **same** sphere, we shall have a conformally-distorted image of the original gnomonic grid in the vicinity of the 'north pole', but with a resolution enhanced by the factor S.



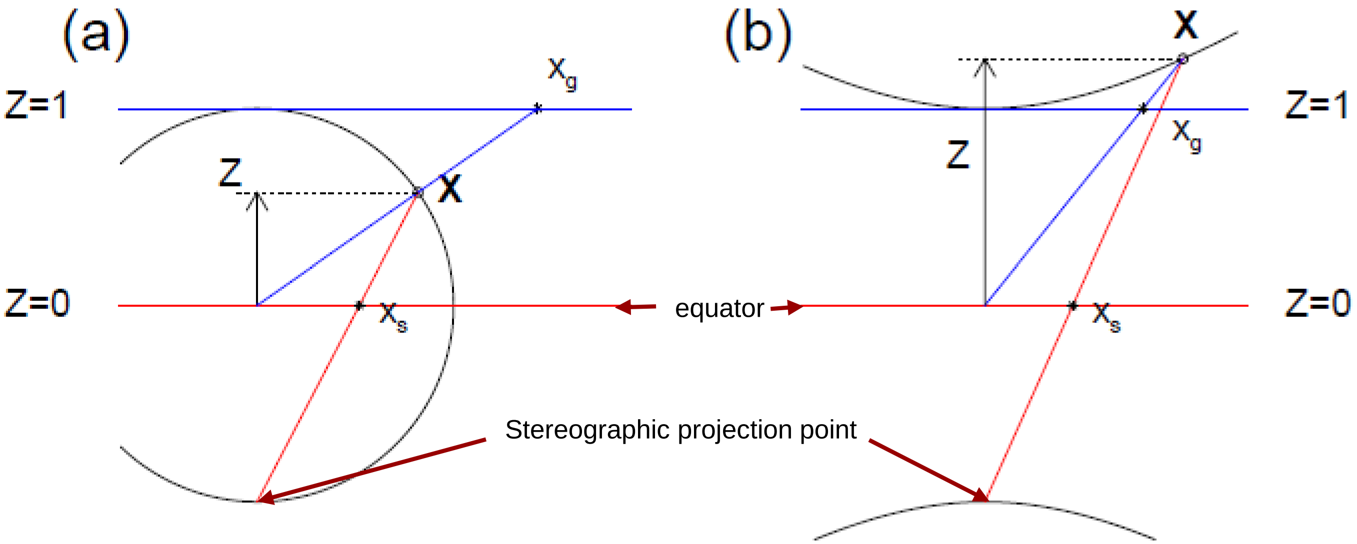
But an equivalent picture is that the sphere on which the original gnomonic grid is constructed is of radius 1/S, and its stereographic projection onto the equatorial plane is **NOT scaled** before its back-projection onto the unit-radius 'Earth'.

What if we replace parameter S by K= S^2?

When 1/S is the spherical radius, K=S^2 is the **GAUSSIAN CURVATURE**.
But there is no obstacle to allowing K to be a negative value!
For negative curvature, K, the original ‘sphere’ is actually a ‘pseudo-sphere’ or ‘hyperbolic plane’.

Remarkably, gnomonic grids, with geodesic lines, and stereographic projections still occur.

But we are now restricted (by topology) to **Limited Area** domains



Gnomonic and stereographic projections illustrated for a true sphere of curvature K=+1

Gnomonic and stereographic projections for a pseudo-sphere of curvature K = -1

Remarks

The conformal mappings of a sphere to itself to enhance the resolution of a global model locally, with enhancement factor, S>0, were proposed by F. Schmidt in 1977.

When the sphere is represented by some standard stereographic projection to the complex plane, then this continuous group of mappings are known as ‘Mobius transformations’.

By replacing the parameter by K=S^2 ,the traditional Schmidt mappings are retained for the K>0 range, but an extended range of parameter space, K<0, is now opened up which, while not valid for the whole globe, is certainly valid for mapping gnomonic grids, with their line-spacing parameter, B, to limited areas.

However, to regularize the otherwise singular behavior of the mapping transformations near the special stereographic cases, K=0, we need to rescale the original line-spacing parameter, B, which we do by replacing it by:

A=KB.

Thus, our final parameter pair, defining the limited area Extended Schmidt Gnomonic (ESG) grid, uses the parameter pair, **(A,K)**.

Optimizing the map parameters

In order to apply the extended Schmidt-transformed gnomonic mapping in an optimal way for a limited area domain we must first define an objective optimality criterion.
The Jacobian matrix of the mapping at a point, relating the rate of change of the Earth-centered Cartesian 3-vector, **X**, with respect to the changes in the components of the map's coordinate 2-vector, **x**, is defined,

$$J_{i,j}(x) = \frac{\partial X_i}{\partial x_j}.$$

An associated symmetric 2x2 GRAM MATRIX is defined:

$$G(x) = J^T(x)J(x).$$

Our objective should be to minimize some kind of 'variance' of G, since this can serve as an objective measure of the map's overall inhomogeneity.

Quantifying the inhomogeneity of a map

The departure of the Gram matrix at each point from a constant multiple of the identity is a measure of the map's local deformation, or anisotropy. We could seek to minimize some integrated squared-measure of the departure of the the Gram matrix from a constant, but there is another important diagnostic we should also consider: the variability of the areal resolution, which is related to the determinant of G. However, formulating the diagnostics of G directly is not as satisfactory as taking the diagnostics from the matrix logarithm, L, of G.

$$\boldsymbol{L}(\boldsymbol{x}) = \ln\left(\boldsymbol{G}^{\frac{1}{2}}(\boldsymbol{x}) \right) = \frac{1}{2}\ln(\boldsymbol{G}(\boldsymbol{x}))$$

Rescaling the map coordinates has an additive effect of L, so we can redefine L by subtracting the mean constant matrix part of it, and define a parameterized diagnostic Q of inhomogeneity:

$$Q_\gamma = \frac{\iint (1-\gamma)\mathrm{trace}(L(x)^2)+\gamma(\mathrm{trace}(L(x)))^2 dx_1 dx_2}{A}$$

Where the map-space area, *A*, of the rectangular domain is just:

$$A = \iint dx_1 dx_2.$$

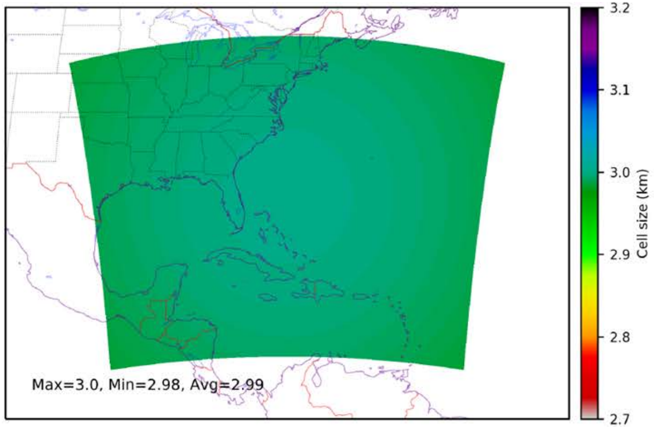
When the parameter, γ, vanishes, the diagnostic, Q_γ, is measuring the variance of ALL components of inhomogeneity of the mapping equally; when $\gamma > 0$, it gives extra weight to the inhomogeneity of the areal resolution. (Note, $\gamma < 1$ always.)

We typically choose a value, γ=0.8.

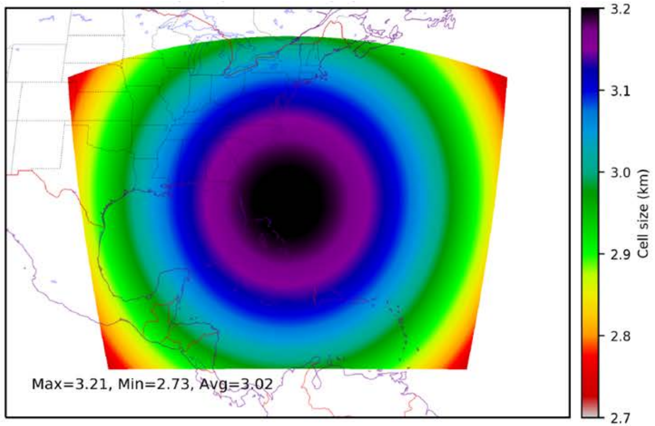
Grid parameter optimization minimizes Q_γ with respect to parameters (A,K).

Generally, this leads to a NEGATIVE optimal K.

Contours of grid cell size show the advantages of extending the parameter space



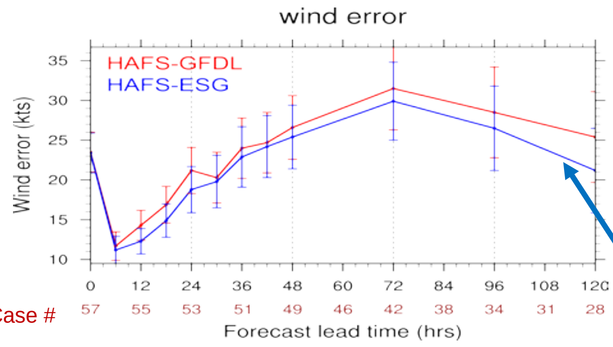
Extended Schmidt gnomonic grid

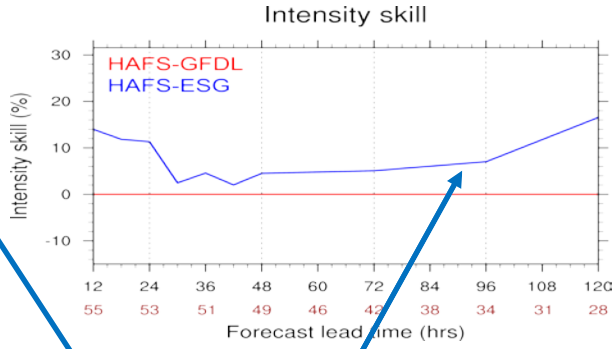


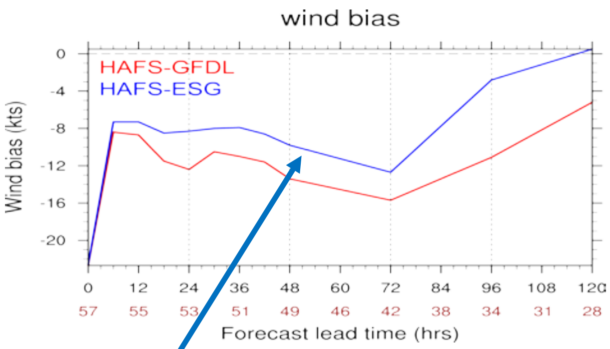
Ordinary gnomonic grid

(Figures kindly provided by Chan-Hoo Jeon)

ESG vs. GFDL grids for HAFS (Hurricane Analysis and Forecast System)-SAR (Stand-Alon Regional model)





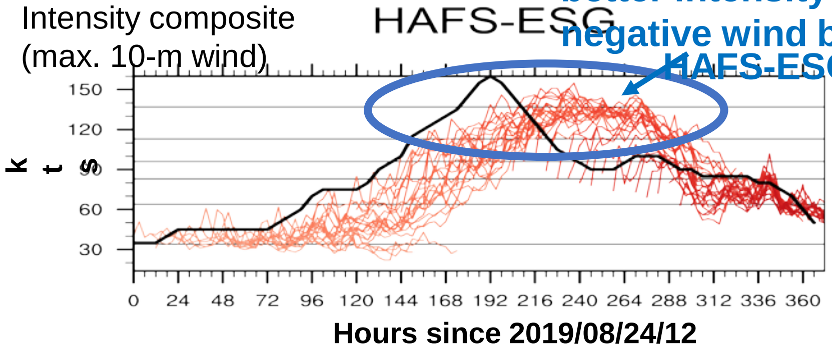


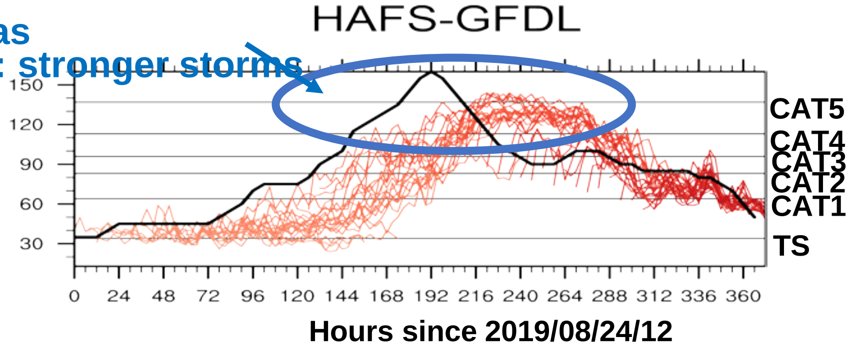
HAFS-ESG: smaller intensity error, better intensity skill and reduced negative wind bias

Hurricane Dorian 2019

Intensity composite (max. 10-m wind)

HAFS-ESG: stronger storms





- Track forecasts close to each other
- ESG improve on intensity forecast and reduce negative wind bias
- Size similar; ESG tend to reduce size error in longer lead times
- ESG predicted stronger TC: more cat4 and cat5 (Slide taken from Jili Dong's presentation at this meeting)

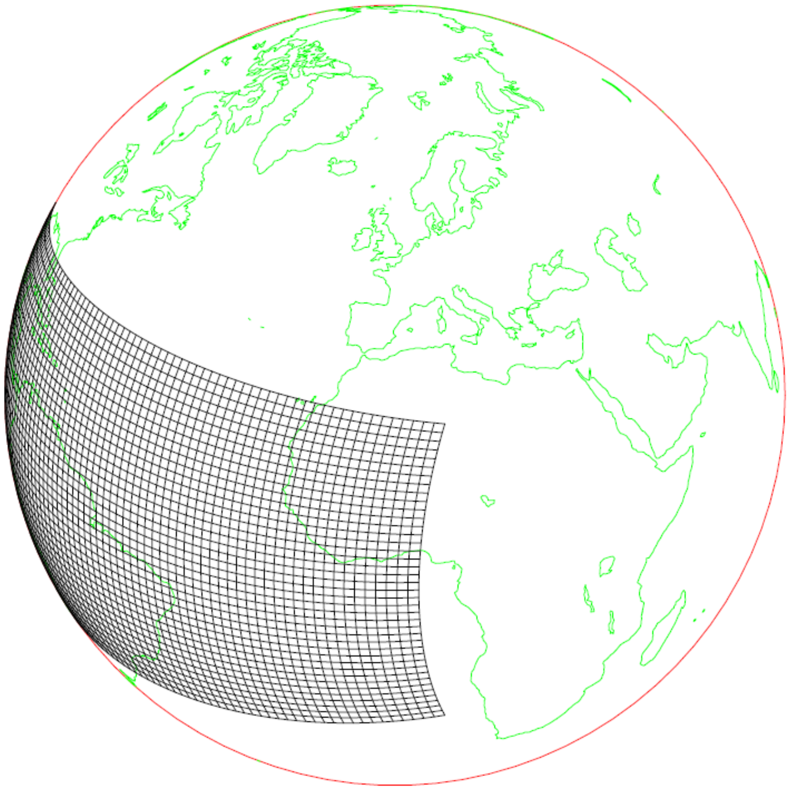
Remarks

The ESG grid allows rectangular domains of very large geographical dimensions without excessively large disparities of resolution.

The generally negative optimal **K** implies a slight relative 'flaring' of each grid, at its edges, compared to the ordinary family of gnomonic grids, whose corners are contrastingly blunt (obtuse angles).

The pinched (acute angle) corners of fairly large ESG grid can be seen in figure to the right.

The **A** parameter is of either sign, depending on the domain shape and size.



Conclusion

We have found a way to extend the existing space of two parameters of the FV3-cubed sphere Schmidt-Gnomonic grids by a form of analytic continuation so that the old parameter domain remains a subset of the new extended one.

The extension of the Schmidt parameter, whereby 'imaginary S' becomes a valid selection (i.e., negative K) ONLY applies to limited domains.

But, for limited domains, the extension to negative K allows surprisingly homogeneous-resolution grids to be found over very large regions.