Observation error in (for) data assimilation

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Topics

- The role of observation error in data assimilation.
- How we interpret observation error.
- Traditional and more recent methods for estimating it for/with DA systems.

The statistical analysis equation

With several conditions met, the update step in nearly all data assimilation systems can be written:

Scalar:

$$a = b + \frac{\sigma_b^2}{\sigma_b^2 + \sigma_o^2}(o - b)$$

Vector/matrix:

$$\mathsf{x}^{\mathsf{a}} = \mathsf{x}^{\mathsf{b}} + \frac{\mathsf{BH}^{\mathsf{T}}}{\mathsf{HBH}^{\mathsf{T}} + \mathsf{R}}\left(\mathsf{o} - \mathsf{Hx}^{\mathsf{b}}\right)$$

$a \in \mathsf{x}^{\mathsf{a}}$	$b \in \mathsf{x}^{\mathsf{b}}$	$\sigma_b^2 \in \mathsf{B}$	$\sigma_o^2 \in \mathsf{R}$	$o \in \mathsf{o}$	H
Analysis	Guess	Bkgd error	Obs error	Obs	$o = h(x)$

All rolled up in σ_o^2?

- Observation error should probably include:
  ◦ Representativeness error with respect to my model (grid spacing and interpolation, other errors in H).
  ◦ Representativeness error with respect to my observation (temporal and spatial sampling).

- In data assimilation, the definition may be slightly different:
  ◦ We don't know exactly what an instantaneous model forecast means in terms of truncation and with respect to numerical diffusion.
  ◦ We assume unbiased observations $\langle \varepsilon_o \rangle = 0$ so we don't want to include it in our estimates of observation error.

- Errors in B and R are easily confused when so many sources of error are floating around.

Data to exploit

Innovations are the primary information source:

$$o - b, \; o - a$$

Data assimilation perspective:

- Both short-term prediction and observation errors contribute (the crux).
- In some ways, we don't care whether the observation error specification is correct, and in some assimilation systems it is explicitly used to offset other sources of error.
- One way to state the goal is that we want to produce the best agreement between background error and the sum of background and observation errors. The indicates that the system is working near optimally.

A classic approach

Gandin (1963)
Rutherford (1972)
Hollingsworth and Lönnberg (1986)

Assumptions:

- Forecast (background) error correlations are isotropic and homogeneous.
- Observation errors are uncorrelated $\left\langle \varepsilon_{oi}, \varepsilon_{oj} \right\rangle = 0$.
- Forecast and observation errors are uncorrelated $\langle \varepsilon_o, \varepsilon_b \rangle = 0$.

A classic approach

Technique:

- Compute correlation between all possible pairs of innovation time series at non-zero separation distance.

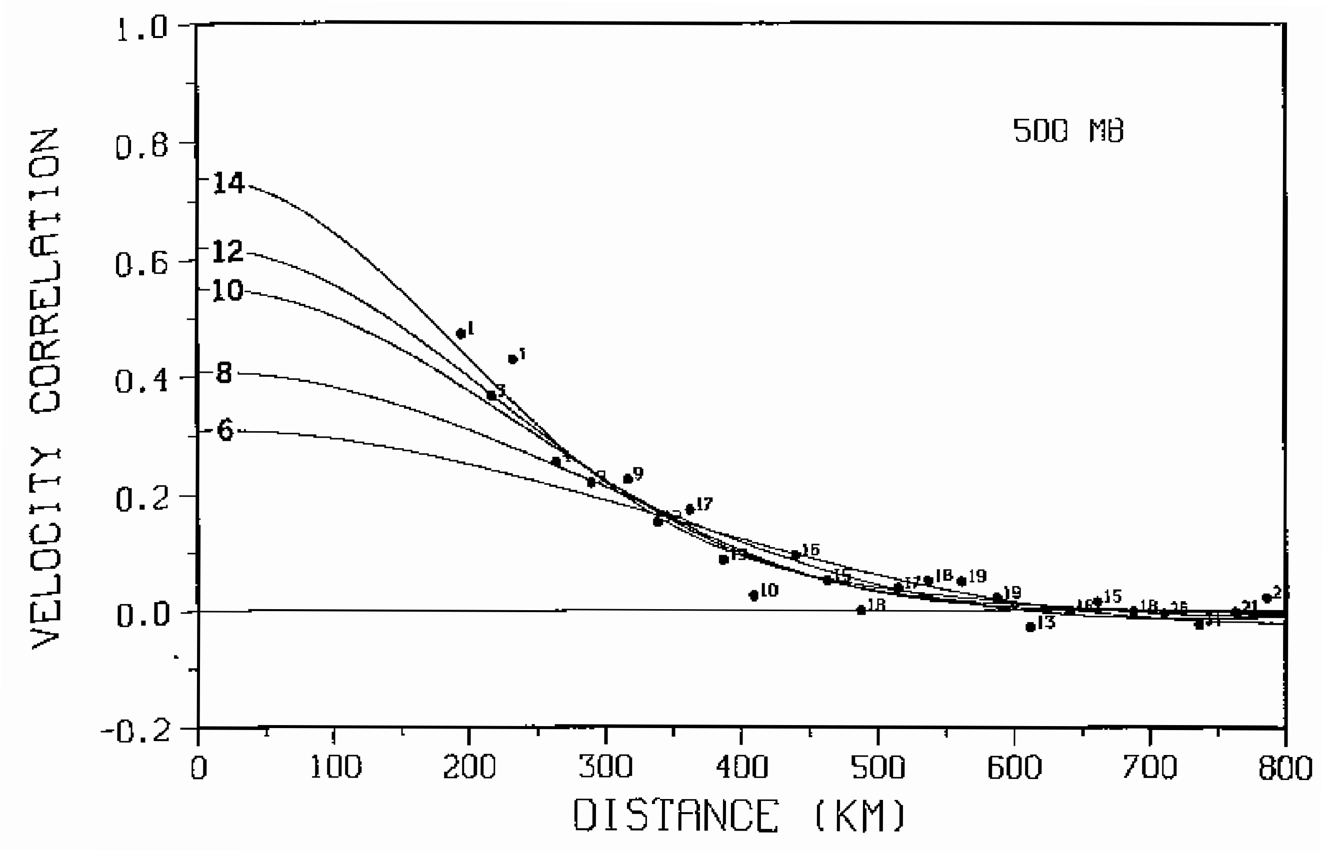
- Bin by distance and average the bin, then fit a correlation function and extrapolate it to 0 separation distance.
  ∘ Expand the exponential correlation function into a truncated series (e.g. Fourier)
  ∘ Truncation is at a wavenumber unresolved by the grid

- Extrapolate to zero separation distance.

Interpretation: $$C(0) = \frac{\sigma^2_{resolved}}{\sigma^2_{resolved} + \sigma^2_{subgrid} + \sigma^2_{instrument}}$$

A classic approach

$$\sigma^2_{subgrid} + \sigma^2_{instrument} = \frac{1-C(0)}{C(0)}\sigma^2_{resolved}$$



DA system evaluation

Desroziers et al. (2005) and others

- Background (forecast) spread and error should match for an optimal solution
- Indicates a well-tuned system
- Assumes the forward operator H is perfect

$$
\begin{aligned}
d &= o - b \ or \ o - a \\
\langle d_b, d_b \rangle &= \langle x_b, x_b \rangle + \left\langle \varepsilon_o^2 \right\rangle \\
\cdots & \\
\langle d_a, d_b \rangle &= \left\langle \varepsilon_o^2 \right\rangle
\end{aligned}
$$

DA system evaluation

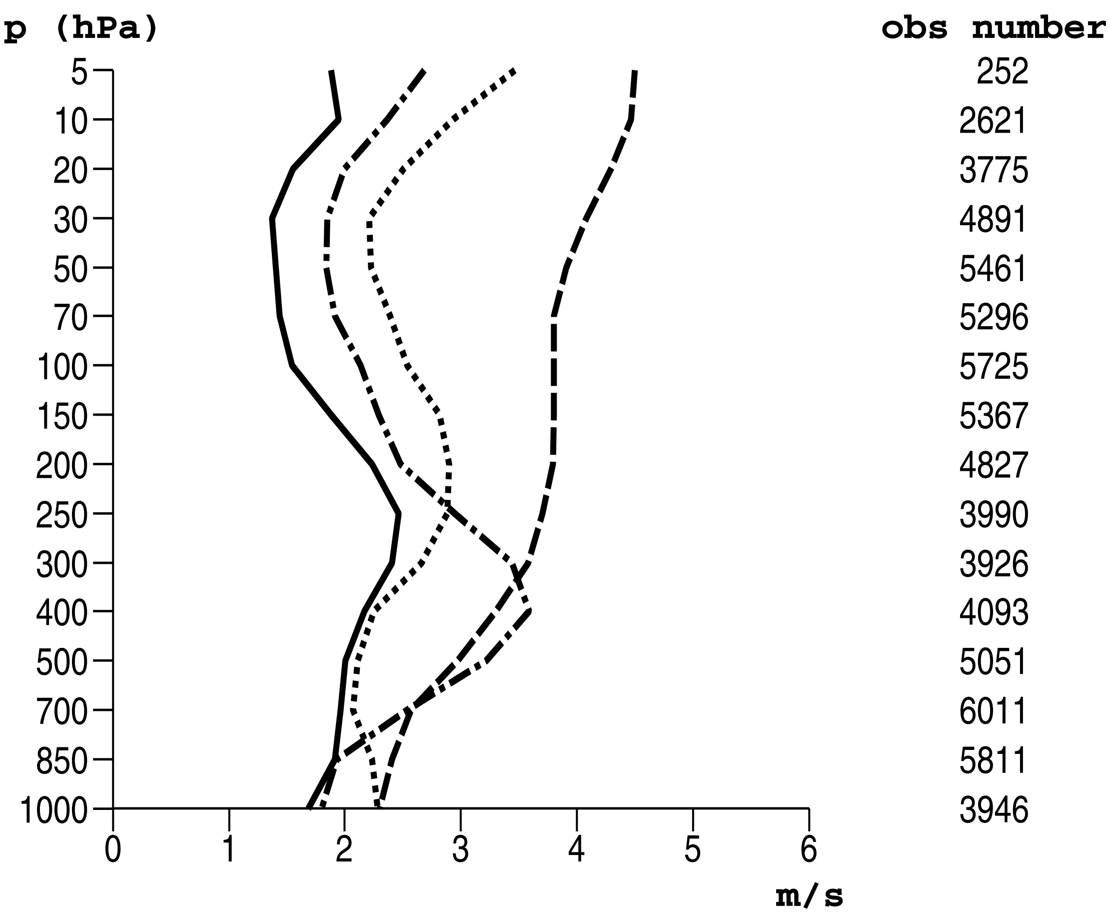


Figure 2. Vertical profiles of diagnosed square roots of background (solid) and observation (dotted) error variances for radiosonde wind observations in the northern hemisphere, compared with profiles for corresponding background (dash-dotted) and observation errors (dashed). All values are in m s^{-1}. The numbers of observations used to compute statistics are shown on the right.

Desroziers et al. (2005) *QJ*

Further heuristic evidence from DA

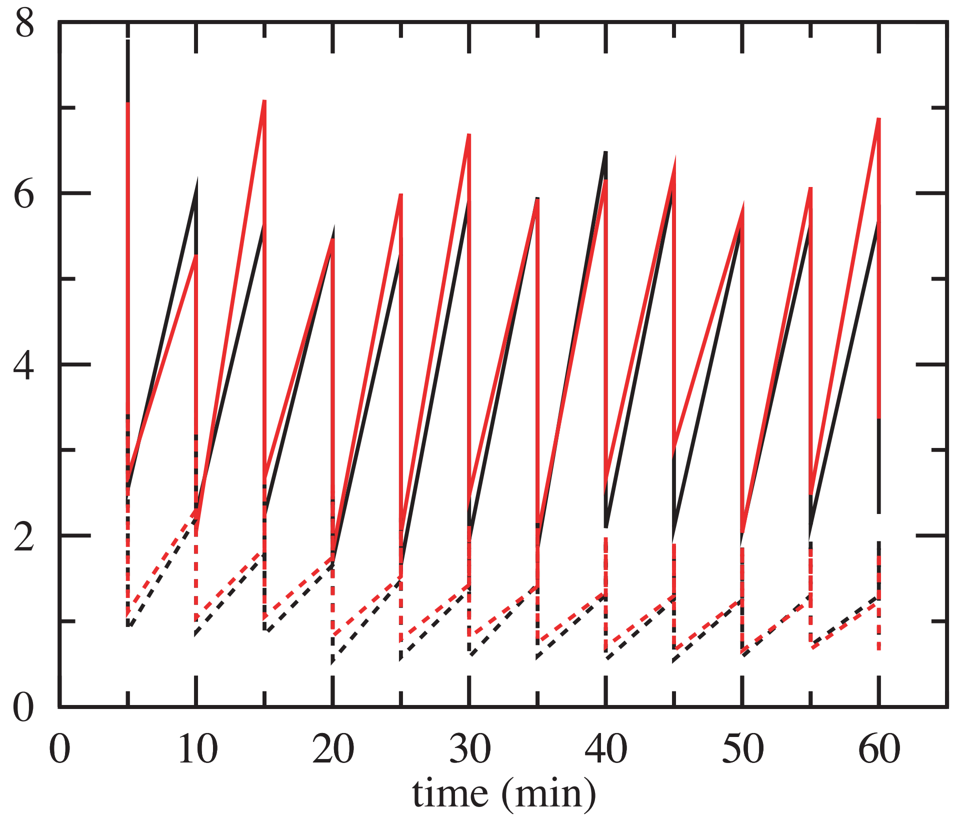
- In ensemble data assimilation, typically

$$\langle d_b, d_b \rangle > \langle x_b, x_b \rangle + \left\langle \varepsilon_o^2 \right\rangle$$

  If you believe that your model is good and your assimilation system is effective, then one interpretation is that ε_o is assigned too small.

- In incremental data assimilation (e.g. ensemble filters, 3DVar), rapid error growth from analysis to background can suggest imbalances introduced by fitting the observations too closely. The interpretation is that ε_o is assigned too small.

Example



Error (solid) is much greater than spread (dashed). Colors are two different samples. Sawtooth pattern is typical of incremental data assimilation systems. Courtesy Tong and Xue (2008); Doppler wind assimilation.

Summary/comments

- Model/DA systems can provide information about observation error estimates that are appropriate for that model/DA system, but not in general.

- The definition of observation error may be different from what verification efforts require.

- In DA, the many sources of error are difficult to untangle and interpret, and observation error can be used to accomplish more than one job.