Radiance Data Assimilation

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Outline

• Introduction
• Different types of satellite data.
• Basic Concepts for Assimilating Observations from Passive Nadir Sounders
• Assimilating satellite radiances.
  – Data assimilation equation
  – Quality control and Observation Errors.
  – Bias correction.
  – Thinning
  – Monitoring.
• Some Comments on Cloudy Radiances.
• Final Comments
Introduction

• Satellites instruments do not directly measure the atmospheric state.
• Instead they measure radiation emitted by and/or transmitted by the atmosphere.
• This presentation describes the relationship between the atmospheric state and the observed radiation. And how the information contained therein is exploited through assimilation into the NWP model.
Different Types of Satellite Data
Meterological Satellite Constellation (from WMO)

- Aqua QuickScat TRMM
- GOES-R (USA) 75W
- GOES-R (USA) 135W
- METEOR 3M (Russian Federation)
- FY-1/3 (China)
- FY-2/4 (China) 105E
- GMS-5/MTSAT-1R (Japan) 140E
- COMSAT-1 (Rep of Korea) 120E
- GPM ADEOS II GCOM
- ENVISAT/ERS-2 METEOR 3M N1 SPOT-5
- MSG (EUMETSAT) 0 Longitude
- METEOSAT (EUMETSAT) 63E
- GOMS (Russian Federation) 76E
- INSATs (India) 83E
- Metop (EUMETSAT)
- NPOESS (USA)

Other R&D oceanographic land use atmospheric chemistry and hydrological missions

Terra NPP Jason-1 Okean series
Different Types of Satellite Data

- Active (bouncing a signal off something)
  - Wind Lidar
  - SAR
  - Cloud radar
  - Scatterometry
Different Types of Satellite Data

• Occultation (signal passing through atmosphere)

HALOE
SAGE
SCIAMACHY
Different Types of Satellite Data

- Passive (receiving radiative signal from source)

  Visible instruments
  IR instruments
  Microwave instruments
Passive Instruments

• This talk will focus on passive infrared and microwave instruments as they are the most common and biggest contributors to Numerical Weather Prediction
Geometry: Limb vs Nadir Sounding

- Limb sounding: Viewing the Earth’s atmosphere tangentially
  - Higher vertical resolution
  - Lower horizontal resolution
  - Most often used for observing the stratosphere and above.
  - Not often used in NWP
Geometry: Limb vs Nadir Sounding

- Nadir sounding: Viewing towards the Earth’s surface
  - Lower vertical resolution
  - Higher horizontal resolution
  - Most often used in NWP
Currently operationally Assimilate radiances at NCEP
Basic Concepts for Assimilating Observations from Passive Nadir Sounders
Taking advantage of the frequency dependent atmospheric absorption

- The atmosphere is variously opaque and transparent to electromagnetic radiation depending on the wavelength.
- We take advantage of this, plus the fact that at longer wavelengths we can observe thermal emission from the atmosphere itself to infer information on the atmosphere’s temperature and humidity profile.
Atmospheric Opacity in the Microwave Spectrum
An Infrared Spectrum

Dashed lines are Planck curves for 200K-300K at 20K intervals.

Radiance (mW/cm²/sr/cm⁻¹)

Wavenumber (cm⁻¹)

CO₂, O₃, H₂O, CO₂

Longwave window

Shortwave window
An Infrared Spectrum
- converted into Brightness Temperature

Shortwave window
(with solar contribution)

Longwave window

Wavelength (µm)

Brightness Temperature (K)

\[ \text{CO}_2, \text{H}_2\text{O}, \text{O}_3 \]
Illustration of Jacobian or Weighting Function
IASI vs HIRS: The Thermal InfraRed
AIRS & HIRS Jacobians in the 15µm CO$_2$ band

100 hPa

1000 hPa

Jacobian (dBRT/dT)

Pressure hPa
Forward models

• To exploit these radiances, it is important to have an accurate way of simulating them from the atmospheric state.

• Line-by-line (LBL) models use state-of-the-art spectroscopic databases to make these calculations at high spectral resolution.

• These monochromatic calculations are then combined using the instruments’ spectral response functions (ISRFs) to simulate what the instrument observes.

• This can be very slow. Too slow for operational radiance assimilation.
Fast Forward models

• To allow radiances to be operationally assimilated, fast radiative transfer models, which use regression schemes to simulate the output from LBL models, have been produced.

• The two main fastmodels used operationally in NWP centers are RTTOV (developed by the EUMETSAT NWPSAF) and CRTM (JCSDA).

• The errors in the fastmodel are not usually a significant component of the total error budget.

• Most importantly, fastmodels allow the Jacobians (and the model adjoint) to be calculated efficiently.
RT models also need to know the surface emissivity

• Over ocean we usually have models, e.g.:
  – ISEM (infrared)
  – FASTEM (microwave)

• Over land we often use atlases, either of the emissivities themselves or of the land type.

• Emissivities can also be retrieved from the observations themselves.
Surface Emissivity Infrared
Emissivities over the ocean are very low in the microwave.
• So we have observations of the radiation emitted from the atmosphere at various frequencies corresponding to emission and absorption at various levels

• Now what?
• Unless we can infer the temperature profile we won’t be able to do much else.
• To do this we need to choose frequencies where we know the absorption profiles already.
• We choose gases with a constant distribution to do this.
• For the infrared we use $\text{CO}_2$.
• For the microwave we use $\text{O}_2$.
• These are hence known as *temperature sounding bands*.
• But all bands are sensitive to temperature, often – as in the case of $\text{H}_2\text{O}$ – with sharper Jacobians.
• Once we have a good temperature profile we can use that to infer molecular abundances of variable species using appropriate frequencies.
  – This is actually performed simultaneously with the temperature estimation when we do data assimilation.
Obtaining vertical profiles

The Jacobians give the sensitivity to the vertical profiles of temperature/gases/clouds etc.

If we sum the contribution of each channel, we can get a very accurate estimate of the mean atmospheric temperature (with very low vertical) resolution.

If we take differences between each of the channels we can infer the profile with high vertical resolution, but the result will be very noisy.

When we assimilate the radiance observations we are effectively producing a minimum variance solution to the problem: which is a compromise between these two extremes.
Assimilating satellite radiances

Data Assimilation Equation
Atmospheric Analysis Problem

\[ J = J_b + J_o + J_c \]

\[ J = (x-x_b)^T B_x^{-1} (x-x_b) + (y-K(x))^T (E+F)^{-1} (y-K(x)) + J_C \]

\[ J = \text{Fit to background} + \text{Fit to observations} + \text{constraints} \]

\[ x = \text{Analysis} \]

\[ x_b = \text{Background} \]

(usually a short-range forecast from the previous cycle)

\[ B_x = \text{Background error covariance} \]

\[ K = \text{Forward model (nonlinear)} \]

\[ O = \text{Observations} \]

\[ E+F = R = \text{Instrument error} + \text{Representativeness error} \]

\[ J_C = \text{Constraint term} \]
Atmospheric Analysis Problem

\[ J = J_b + J_o + J_c \]

\[ J = (x-x_b)^T B_x^{-1} (x-x_b) + (y-K(x))^T (E+F)^{-1} (y-K(x)) + J_C \]

The difference between the observations and the background transformed into model space, the first guess departure, is an important measure. It is often the basis of quality control procedures.

- \( x \) = Fit
- \( x_b \) = Background
- \( B_x \) = Background error covariance
- \( K \) = Forward model (nonlinear)
- \( O \) = Observations
- \( E+F \) = Instrument error + Representativeness error
- \( J_C \) = Constraint term
Assimilating satellite radiances

Quality Control
Quality Control Procedures

- The quality control step may be the most important aspect of satellite data assimilation.
- Data which has gross errors or which cannot be properly simulated by forward model must be removed.
- Most problems with satellite data come from 4 sources:
  - Instrument problems.
  - Clouds and precipitation simulation errors.
  - Surface emissivity simulation errors.
  - Processing errors (e.g., wrong height assignment, incorrect tracking, etc).
Quality Control Procedures

• IR cannot see through most clouds.
  – Cloud height difficult to determine – especially with mixed FOVs.
  – Since deep layers not many channels completely above clouds.
• Microwave impacted by clouds and precipitation but signal is smaller from thinner clouds.
• Surface emissivity and temperature characteristics not well known for land/snow/ice.
  – Also makes detection of clouds/precip. more difficult over these surfaces.
• Error distribution may be asymmetric due to clouds and processing errors.
Effect of Cloud on IR Spectrum

- No Cloud
- Mid-Level Cloud
- High Opaque Cloud
- H$_2$O
- CO$_2$
- O$_3$
- Noise
Assume the cloud is a single layer at pressure $P_c$ and with unit emissivity and coverage within the FOV, $N_c$.

- $0 \leq N_c \leq 1$
- $P_c$ is below the tropopause and above the ground
- Find $P_c$ and $N_c$ so that the RMS deviation, $J(N_c, P_c)$, of the calculated cloud from the model (over a number of channels) is minimized.
- Remove all channels that would be radiatively affected by this cloud.

\[ R_{\text{cld}}(v, P_c) = N_c R_{\text{overcast}}(v, P_c) + (1-N_c) R_{\text{clear}}(v, P_c) \]

\[ J(N_c, P_c) = \sum_v \left( \frac{R_{\text{cld}}(v, P_c) - R_{\text{obs}}(v)}{\sigma(v)} \right)^2 \]

$\sigma(v)$ is the assumed observation error for channel $v$.

This calculation should be done in radiance, not brightness temperature.
Cloud detection in the infrared

A non-linear pattern recognition algorithm is applied to departures of the observed radiance spectra from a computed clear-sky background spectra.

This identifies the characteristic signal of cloud in the data and allows contaminated channels to be rejected.
Atmospheric Opacity in the Microwave Spectrum

![Graph showing optical depth vs. frequency (GHz) for different channels and gases.](image-url)
Cloud detection in the microwave

Water sensitivity results in a gradient around 0.5

Cloud sensitivity results in a gradient greater than 1.0

Used Obs deemed Cloudy

Used Obs deemed Clear
(unused obs are small dots)
Observational Errors

• Observation errors specified based on instrument errors and statistics
• Generally for satellite data, variances are specified a bit large since the correlated errors (from RT and instrument errors) are not well known.
• Observation errors are also generally specified as being uncorrelated spectrally, but efforts are being made to determine the off-diagonal components of the observation error covariance matrix.
## Satinfo File

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- **Use Channel?**
- **Assigned**
- **Maximum allowed FG Departure (after bias correction)**
- **Observation Error**
Assimilating satellite radiances

Bias Correction
Bias Correction

• The differences between simulated and observed observations can show significant biases.
• The source of the bias can come from:
  – Inadequacies in the characterization of the instruments.
  – Deficiencies in the forward models.
  – Errors in processing data.
  – Biases in the background.
• Except when the bias is due to the background, we would like to remove these biases.
Bias Correction

• Currently bias correction only applied to a few data sets:
  – Radiances.
  – Radiosonde data (radiation correction and moisture).
  – Aircraft data.

• For radiances, biases can be much larger than signal. Essential to bias correct the data.

• NCEP currently uses a 2-step process for radiances (other centers are similar).
  – Angle correction: Very slowly evolving; different correction for each scan position. Specified in `satbias_angle` file
Scan dependent biases for AMSU

platform: amsua
region: global
variable: observed—simulated (without bias correction) (K)
valid: 00Z 20FEB2001 00Z 22MAR2001

channel 5

channel 6

channel 7

channel 8

satellite zenith angle (degrees)
Satellite radiance observations

Bias correction

• Air mass prediction equation for bias – variational bias correction
  – Add to control vector (analysis variables \( x_{n+i} \))

\[
\text{where total bias correction} = \sum_{1}^{n} x_n + i p_i
\]

– Predictors \((p_i)\) for each channel
  • mean
  • path length (local zenith angle determined)
  • integrated lapse rate
  • \((\text{integrated lapse rate})^2\)
  • cloud liquid water
NOAA 18 AMSU-A
No Bias Correction

channel 7
κ 0.3765
f 54.94 GHz
λ 5456.69 μm
avg: 1.837
sdv: 0.369

channel 8
κ 0.3955
f 55.50 GHz
λ 5401.64 μm
avg: 1.263
sdv: 0.505
NOAA 18 AMSU-A
Bias Corrected

channel 7
n 0.3765
f 54.94 GHz
λ 5456.69 μm
avg: -0.022
sdv: 0.200

channel 8
n 0.3955
f 55.50 GHz
λ 5401.64 μm
avg: -0.026
sdv: 0.222
Observation - Background Histogram

DMSP15  July 2004: 1 month

- before bias correction
- after bias correction
Application of NWP Bias Correction for SSMIS F18

Using Met Office SSMIS Bias Correction Predictors

O-B Before Bias Correction

O-B Before Bias Correction

O-B After Bias Correction

O-B After Bias Correction

Unbias & Bias Corrected O-B

Ascending Node  Descending Node

Latitude

Global

Global

Dsc

Asc

Dsc

Asc

T (K)

T (K)

T (K)

50
Bias Correction and QC Interact

Bias Correction
Observations are bias-corrected after quality control

Quality Control
Quality control usually uses bias-corrected observations
Assimilating satellite radiances

Thinning
Thinning or Superobbing

- **Thinning**
  - Reducing spatial or spectral resolution by selecting a reduced set of locations or channels.
  - Can include “intelligent thinning” to use better observation.
- **Superobbing**
  - Reducing spatial or spectral resolution by combining locations or channels.
  - Can reduce noise.
  - Includes reconstructed radiances.
  - Can include higher moments contained in data [Purser et al., 2010](#).
  - Can be done with obs or departures, but should be done after QC.
- **Both can be used to address 3 problems:**
  - Redundancy in data.
  - Reduce correlated error.
  - Reduce computational expense.
Five Order of Magnitude Increases in Satellite Data Over Fifteen Years (2000-2015)

Daily Satellite & Radar Observation Count

- Level 2 Radar
  - 210 M obs
  - 125 M obs
  - 100 M obs

Daily Percentage of Data Ingested into Models

- Received Data: 239.5M
- Selected Data: 17.3M
- Assimilated Data: 5.2M

Received = All observations received operationally from providers
Selected = Observations selected as suitable for use
Assimilated = Observations actually used by models
Assimilating satellite radiances

Data Monitoring
Data Monitoring

• It is essential to have good data monitoring.
• Usually the NWP centres see problems with instruments prior to notification by provider (Met Office especially).
• The data monitoring can also show problems with assimilation systems.
• Needs to be ongoing/real time.
• Monitoring reports from most major NWP centers at: http://research.metoffice.gov.uk/research/interproj/nwpsaf/monitoring.html
Quality Monitoring of Satellite Data

AIRS Channel 453 26 March 2007

Increase in SD
Fits to Guess
Quality Monitoring of Satellite Data

NOAA-19 HIRS July 2\textsuperscript{nd} 2013 – Filter Wheel Motor Problems

Initial Problem
When we stopped assimilating

Initial “fix” to instrument
Some Comments on Cloudy Radiances
Cloudy Radiances

- Most of the above discussion concerns the assimilation of radiances unaffected by cloud.
- Currently we are not operationally assimilating cloudy radiances but the GSI contains experimental code for assimilating such radiances in the microwave.
- The next few slides discuss why clouds are both important and difficult for data assimilation…
- …and discusses one aspect of the modifications we are making to assimilate them
Why are clouds important?

- A decade ago almost all assimilation of satellite radiances assumed the scene was clear of clouds.
- Clouds were considered a source of noise that needed to be removed or corrected for.
- This is not because clouds were not important but because they were difficult.
- By ignoring regions affected by cloud we are not considering some meteorological very important areas.
- By selectively assimilating clear radiances we may be biasing the model (representivity issues).
Clouds can be spatially complex

Often we assume a cloud looks like this…

…when they can really look like this

Spatial structure can be below the resolution of the observation, the model or both.
Clouds can be radiatively complex

- The complexity of the impact of clouds on observed spectra varies greatly with type of cloud and spectral region.
- If clouds are transmissive they will tend to have spectrally varying absorption – and hence emission – which depends on phase (water or ice), crystal habit and particle size distribution.
- Scattering from cloud and precipitation particles can be very significant – tends to lower the observed brightness temperature.
Clouds can introduce non-linearities

- The radiative signal from clouds is often large and non-linear so the tangent-linear assumption used in variational data assimilation does not hold.
- Quality control that minimizes the impact of this non-linearity is required.
Clouds need to be consistent with temperature and humidity fields

• Adding clouds to the analysis without ensuring a consistent humidity and temperature profile can be problematic.
  – For example a cloud added into a dry atmosphere will tend to be removed by the model.
The tangent-linear (TL) and adjoint (AD) of full GFS moisture physics are under development and validation.

These linearized moisture physics are added in the minimization to ensure control variables are more physically related and balanced.
Linearized Moisture Physics in the Inner Loop of the Minimization

Initial unbalanced perturbations

More balanced perturbations

T = Temperature  
q = Humidity  
CW = Cloud Water  
CLW = Cloud Liquid Water  
CIW = Cloud Ice Water

CRTM
Linearized Moisture Physics in the Inner Loop of the Minimization - Adjoint

\[
\frac{\partial J}{\partial T^*}, \quad \frac{\partial J}{\partial q^*} \quad \frac{\partial J}{\partial CW^*}
\]

Balanced gradients

\[ J \text{ is the minimization cost function} \]

\[
\frac{\partial J}{\partial T^*}, \quad \frac{\partial J}{\partial q^*} \quad \frac{\partial J}{\partial \text{CLW}^*} \quad \frac{\partial J}{\partial \text{CIW}^*}
\]

Unbalanced gradients

CRTMK

Adjoint of Linearized Moisture Physics

\[ \text{Sum CLW}^* \text{ and CIW}^* \text{ terms} \]
Final Comments
Overall Comments

• Satellite data must be treated carefully.
• Important to be aware of instrument characteristics before attempting to use data.
• No current component of observing system is used “perfectly” or “as well as possible”.
• Computational expense plays important role in design of system.
Questions and/or Comments
Please
Useful References (1/2)

Auligne T.; McNally A. P.; Dee D. P., 207: Adaptive bias correction for satellite data in a numerical weather prediction system, QJRMS, 133, 631-642.


Useful References (1/2)

Dee, D. P., Uppala S., 2009: Variational Bias correction of satellite radiance data in the ERA-Interim reanalysis, QJRMS, 135, 1830-1841.


RTTOV homepage: http://research.metoffice.gov.uk/research/interproj/nwpsaf/rtm/


References


Metop-2 (to be renamed Metop-A after launch) to be launched from Baikonur, Kazakhstan, 30th June 2006. Two more launches in
IASI
Field of view

4 12km pixels/footprint
822km polar orbit
9.30 Equator crossing time
Radiative Transfer : Definitions

- **Radiance**, $I_\nu$, is the radiant energy emitted per unit time, per unit frequency interval, per unit area, and per unit solid angle in a specified direction at a given frequency, $\nu$.
  - The units for radiance are $\text{Wm}^{-2}\text{sr}^{-1}(\text{cm}^{-1})^{-1}$ or equivalent. Radiance is often expressed as the temperature that would produce the equivalent black-body radiance: the *Brightness Temperature*.

- This is not to be confused with *Irradiance* or *Flux Density* which is the total power per unit frequency interval, crossing perpendicular to a unit area which has units $\text{Wm}^{-2}(\text{cm}^{-1})^{-1}$. 
Absorption of radiance in a volume element

Consider monochromatic radiation of frequency $\nu$ passing through a volume element of length $ds$ and cross-sectional area $dA$. It contains a gas with $n$ molecules per unit volume, each with an absorption coefficient of $k_{\nu}$.

If we ignore scattering, the change in radiance across the volume due to absorption is given by:

$$(I_\nu + dI_\nu)dA = -(n k_{\nu} ds) \lambda dA$$

$(n k_{\nu} ds)$ is the absorptivity of the volume element.
Radiative Transfer

Emission of radiance in a volume element

If we continue to ignore scattering, the change in radiance across the volume due to emission from the same volume element is given by:

\[(I_\nu + dI_\nu) dA = \varepsilon_\nu S_\nu dA\]

Where \(\varepsilon_\nu\) is the emissivity of the volume and is \(S_\nu\) the Source Function. In regions of local thermodynamic equilibrium (LTE) – as is usually the case in the troposphere and stratosphere – the Source Function is the Planck Function, \(B(T)\), where \(T\) is the temperature.
Radiative Transfer

Schwarzchild Equation of Radiative Transfer

Combining the terms for emission and absorption gives

\[(I_\nu + dI_\nu)dA = -(n_k_\nu ds) I_\nu dA + \epsilon_\nu S_\nu dA\]

Kirchoff’s Law states that the absorptivity and the emissivity are the same, so \(n_k_\nu ds = \epsilon_\nu\).

If we now define the optical depth, \(\tau\), through \(d\tau_\nu = -n_k_\nu ds\), we obtain the Schwarzchild Equation of Radiative Transfer:

\[
\frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu
\]

As stated above, for LTE, \(S_\nu = B_\nu(T)\), so

\[
\frac{dI_\nu}{d\tau_\nu} = I_\nu - B(T)\nu
\]
Radiative Transfer

Schwarzchild Equation in a scattering atmosphere

\[ \frac{dI_\nu}{d\tau^*} = I_\nu - (1 - \omega_\nu)B - \omega_\nu \int I_\nu(\Omega)p_\nu(\Omega)d\Omega \]

Extinction term
(absorption + scattering)

Emission term

Radiation scattered from all directions, \( \Omega \), into the beam

Now the optical depth, \( \tau^* \nu \), is an extinction optical depth and is defined via
\[ d\tau^* \nu = -n(k_\nu + \sigma_\nu)ds, \]
where is \( \sigma_\nu \) the scattering coefficient.
\( \omega_\nu \) is the single scattering albedo and is given by \( \sigma_\nu / (k_\nu + \sigma_\nu) \).
\( p_\nu(\Omega) \) is the phase function and describes the angular distribution of how incident radiance is scattered.

We are not going to be considering scattering in this talk.
Radiative Transfer

Solution to Schwarzchild Equation

The general solution to the Schwarzchild equation is:

\[ I(\tau_{\downarrow 2}) = I(\tau_{\downarrow 1}) e^{\tau_{\downarrow 2} - \tau_{\downarrow 1}} + \int_{\tau_{\downarrow 1}}^{\tau_{\downarrow 2}} S(\tau) e^{\tau_{\downarrow 2} - (\tau - \tau_{\downarrow 1})} d\tau \]

Radiation at observer

Radiation at lower Boundary (usually the surface for terrestrial planets) ...

... attenuated by the intervening atmosphere

Emission of radiation from the atmosphere itself including reabsorption of this radiation
Radiative Transfer

Transmission and Weighting Functions

The general solution to the Schwarzschild equation is:

\[
I(\tau \downarrow 2) e^{\tau \downarrow 1} \tau \downarrow 2 \cdot S(\tau) e^{\tau \downarrow 1} \tau \downarrow 2 \cdot d\tau
\]

The transmission, between optical depths \(\tau\) and \(\tau_1\), \(T(\tau, \tau_1)\) is \(e^{\tau \downarrow 1} \tau \downarrow 2\) and

\[
dT = -e^{\tau \downarrow 1} \tau \downarrow 2 \cdot S(\tau) \cdot d\tau. \quad \text{So now the solution is:}
\]

\[
I(\tau \downarrow 1) = I(\tau \downarrow 2) T(\tau_2, \tau_1) + \int T_2 \uparrow T_1 \cdot S(\tau) \cdot dT
\]

Which we can transform into pressure, \(p\), coordinates:

\[
I(\tau \downarrow 1) = I(\tau \downarrow 2) T(\tau_2, \tau_1) + \int p_2 \uparrow p_1 \cdot S(\tau) \cdot K(p) \cdot dp
\]

\(K(p) = \partial T(p, p_1) / \partial p\)

is the historical definition of the weighting function as it is the weight given to the source function at each level in the solution.
Radiative Transfer

Weighting Functions and Jacobians

This definition of the weighting function was used in many early retrieval schemes. But in modern data assimilation, the weighting often refers to (e.g., Rogers, 2000) the derivative of the observation (which we now designate $y$) with respect to the state vector $x$. This is the Jacobian matrix, $K = \nabla y(x) \cdot x$.

Here $x$ is typically a vector of temperatures, molecular abundances (including major absorbers such as H$_2$O, CO$_2$, O$_3$, CH$_4$) for many layers throughout the atmosphere, surface properties and often cloud and aerosol properties.
Basic theory of satellite observations

Radiative Transfer
Illustration of Jacobian or Weighting Function
Basic theory of satellite observations

Spectroscopy
So where does atmospheric absorption come from?

- Molecules in the atmosphere have energy stored as rotational, vibrational and electronic components.
- The energy states are quantised and may be transformed through emission or absorption of electromagnetic radiation. This results in discrete spectral emission/absorption features in the spectrum.
  - In the microwave these are due to rotational transitions.
  - In the infrared these are rotational and vibrational transitions.
  - Electronic transitions manifest themselves in the visible and ultraviolet.

Vibrational Modes for CO$_2$
An example of a vibration-rotation band in the infrared CO$_2$ spectrum. Due to considerations of angular momentum, only changes in the rotational quantum number, J, of -1, 0 or 1 are optically active, producing the characteristic three branch structure to the band (some linear molecules have the Q-branch missing).
Line Broadening

• Spectral lines will be broadened through one of the following three processes:
  
  – **Natural broadening:** The finite time of the quantum transition corresponds to an uncertainty in the energy through the uncertainty principle.

  – **Doppler broadening:** Thermal motion of the molecules along the line of sight result in apparent uncertainty in the frequency through the Doppler effect.

  – **Collisional (or pressure) broadening:** Collisions between molecules during emission and absorption results in modification of energy levels and hence broadening of the spectral line.

• In the lower atmosphere, collisional broadening dominates, while Doppler broadening is more important in the upper atmosphere.