Comparison of an analytical and a numerical model for hurricane potential intensity

Richard Rotunno

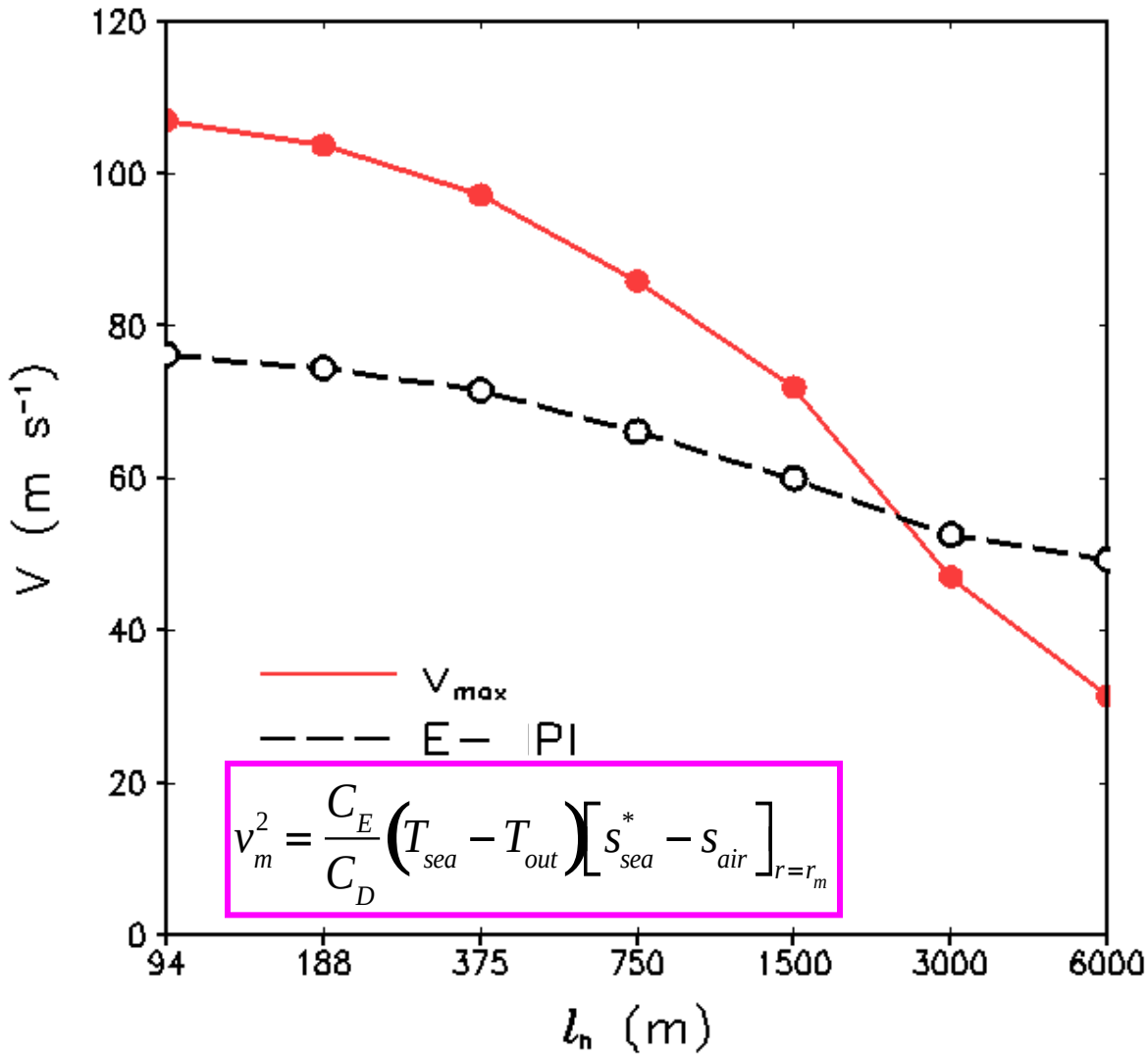
NCAR

Bryan, G. H., and R. Rotunno, 2009: The maximum intensity of tropical cyclones in axisymmetric numerical model simulations. *Mon. Wea. Rev*..

Bryan, G. H., and R. Rotunno, 2009: Evaluation of an analytical model for the maximum intensity of tropical cyclones. *J. Atmos. Sci*.

EMC/MMM/DTC Joint Hurricane Science Workshop

V_{max} , E-PI vs l_h

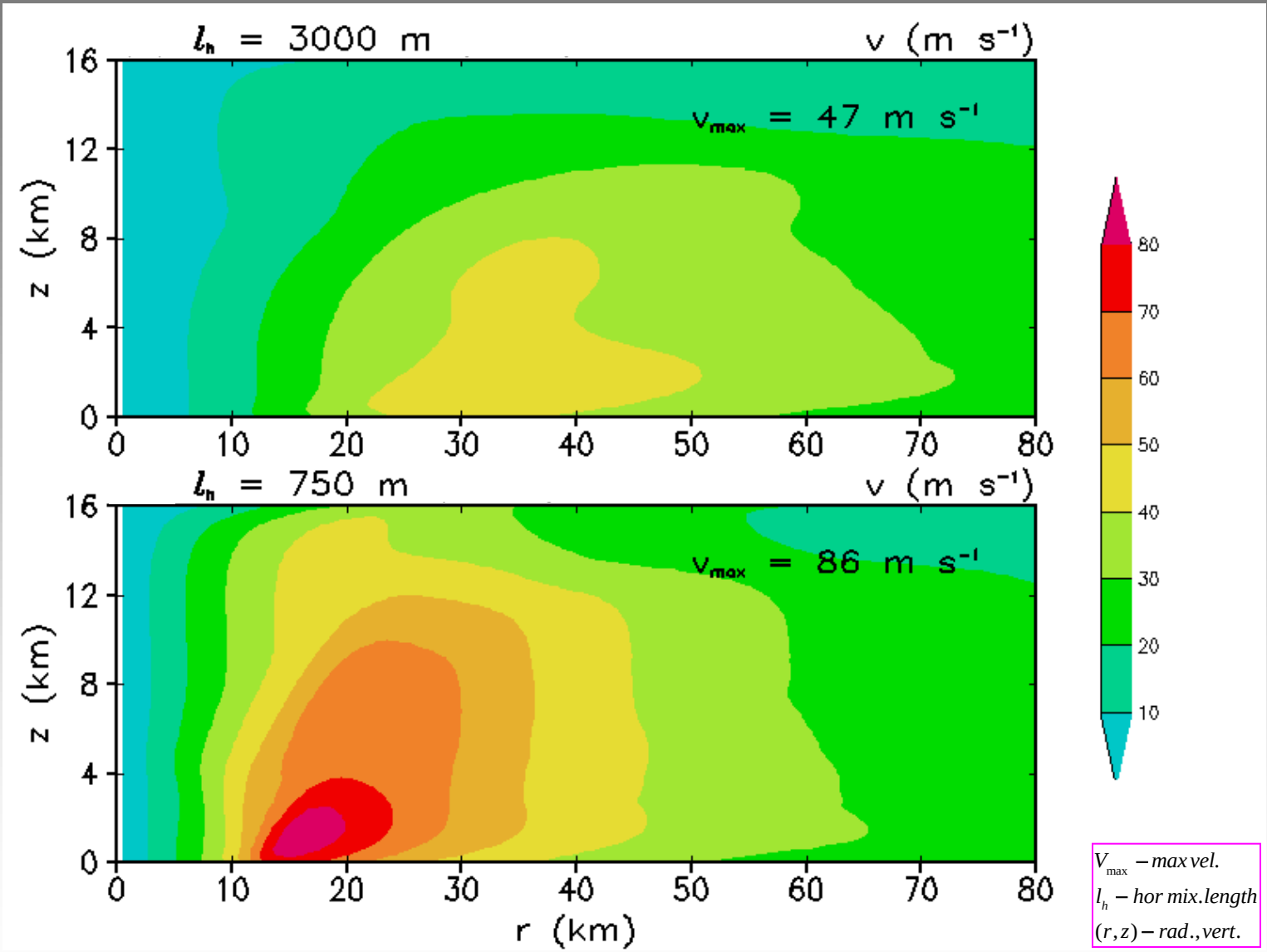


$$v_m^2 = \frac{C_E}{C_D}\left(T_{sea} - T_{out}\right)\left[s^*_{sea} - s_{air}\right]_{r=r_m}$$

V_{max} – max vel.
E - PI - Emanuel Pot. Intensity
l_h – hor mix. length

Bryan and Rotunno (2009 *JAS*)

Sensitivity of Wind Speed to Turbulence Mixing Length l_h



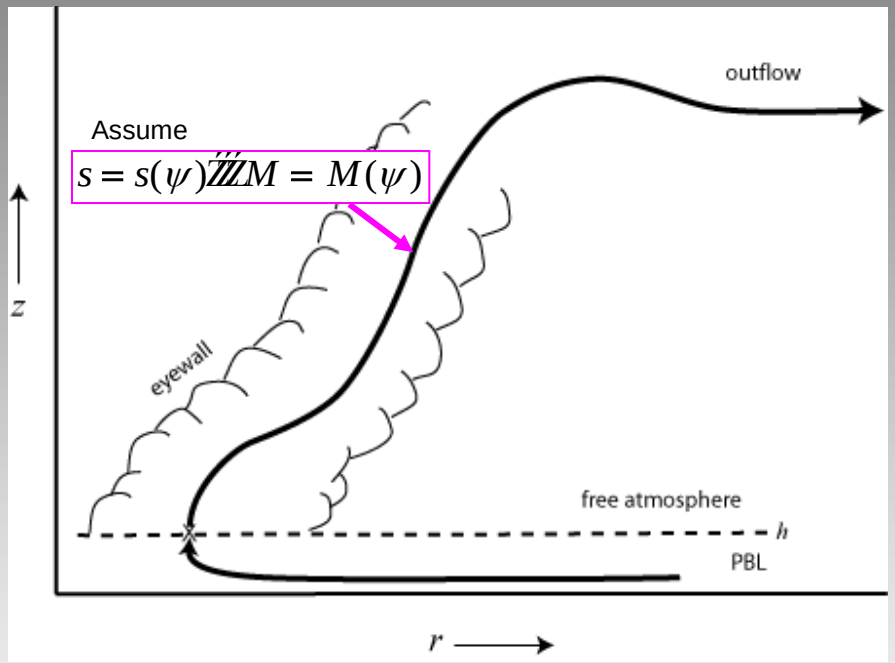
Bryan and Rotunno (2009 *MWR*)

Emanuel Potential Intensity Theory

Steady, inviscid, axisymmetric, pseudo-adiabatic equations of motion projected onto *r-z* plane

$$-\frac{\eta}{\rho_d r} = \frac{1}{2r^2}\frac{dM^2}{d\psi} + T\frac{ds}{d\psi} - \frac{dE}{d\psi} - \frac{f}{2}\frac{dM}{d\psi}$$

v = azi. velocity
η = azi. vorticity
ρ_d = dry air density
T = temperature
M = ang.mom.
E = total energy
f = Coriolis param.
ψ = str. fcn.
s = moist entropy
(r,z) = (rad., alt.)



$r \to \infty$

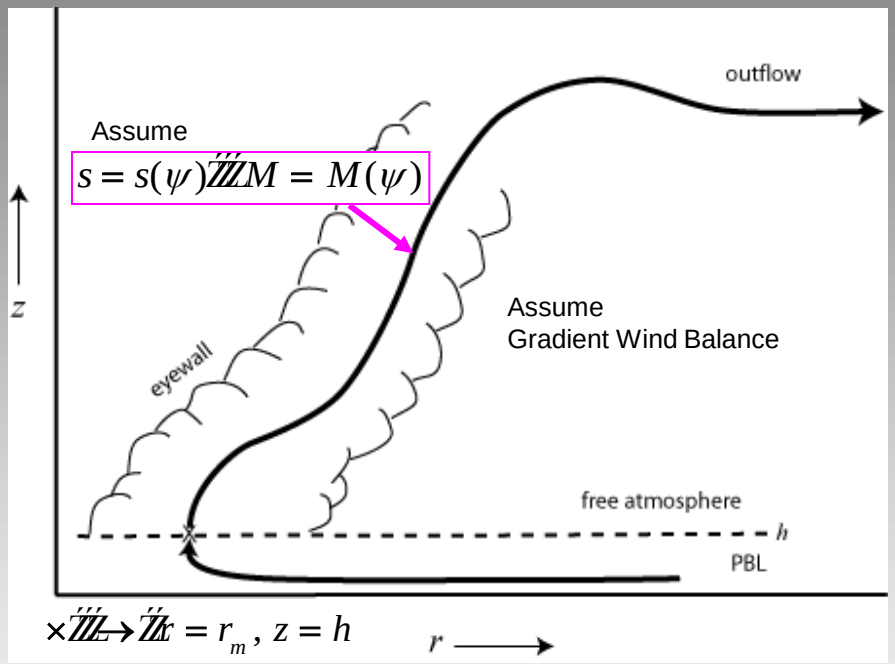
$$0 = T_{out}\frac{ds}{d\psi} - \frac{dE}{d\psi} - \frac{f}{2}\frac{dM}{d\psi}$$

$$-\frac{\eta}{\rho_d r} = \frac{1}{2r^2}\frac{dM^2}{d\psi} + \left(T - T_{out}\right)\frac{ds}{d\psi}$$

Emanuel Potential Intensity Theory

v = azi. velocity
η = azi. vorticity
ρ_d – dry – air density
T – temperature
M – ang.mom.
E – total energy
f – Coriolis param.
ψ – str. fcn.
s – moist entropy
(r,z) – (rad., alt.)

$$r = r_m,\ z = h \longrightarrow -\frac{\eta(r_m,h)}{\rho_d r_m} = \frac{1}{2r_m^2}\frac{dM^2}{d\psi} + \left(T_{sea} - T_{out}\right)\frac{ds}{d\psi}$$



Assume
Gradient Wind Balance

$$0 = \frac{1}{2r_m^2}\frac{dM^2}{d\psi} + \left(T_{sea} - T_{out}\right)\frac{ds}{d\psi}$$

Emanuel Potential Intensity Theory

v = azi. velocity
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ρ_d – dry – air density
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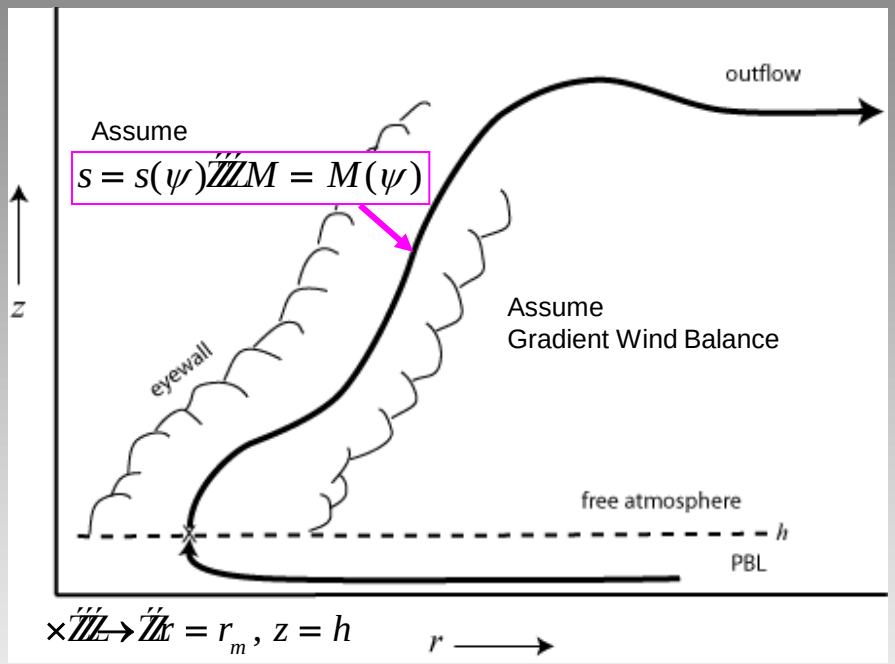
$$\times \frac{d\psi}{dM^2} \longrightarrow \quad 0 = \frac{1}{2r_m^2} + \left(T_{sea} - T_{out}\right)\frac{ds}{dM^2}$$



Assume
Mixed Layer

Assume
Mixed Layer PBL

$$\frac{ds}{dM} = -\frac{\tau_s}{\tau_M}\bigg|_{\substack{z=0\\ r=r_m}} = -\frac{C_E}{C_D}\left[\frac{s^*_{sea} - s_{air}}{rv}\right]_{r=r_m}$$

$$v_m^2 = \frac{C_E}{C_D}\left(T_{sea} - T_{out}\right)\left[s^*_{sea} - s_{air}\right]_{r=r_m}$$

Emanuel (1986 *JAS*)

Components of E-PI

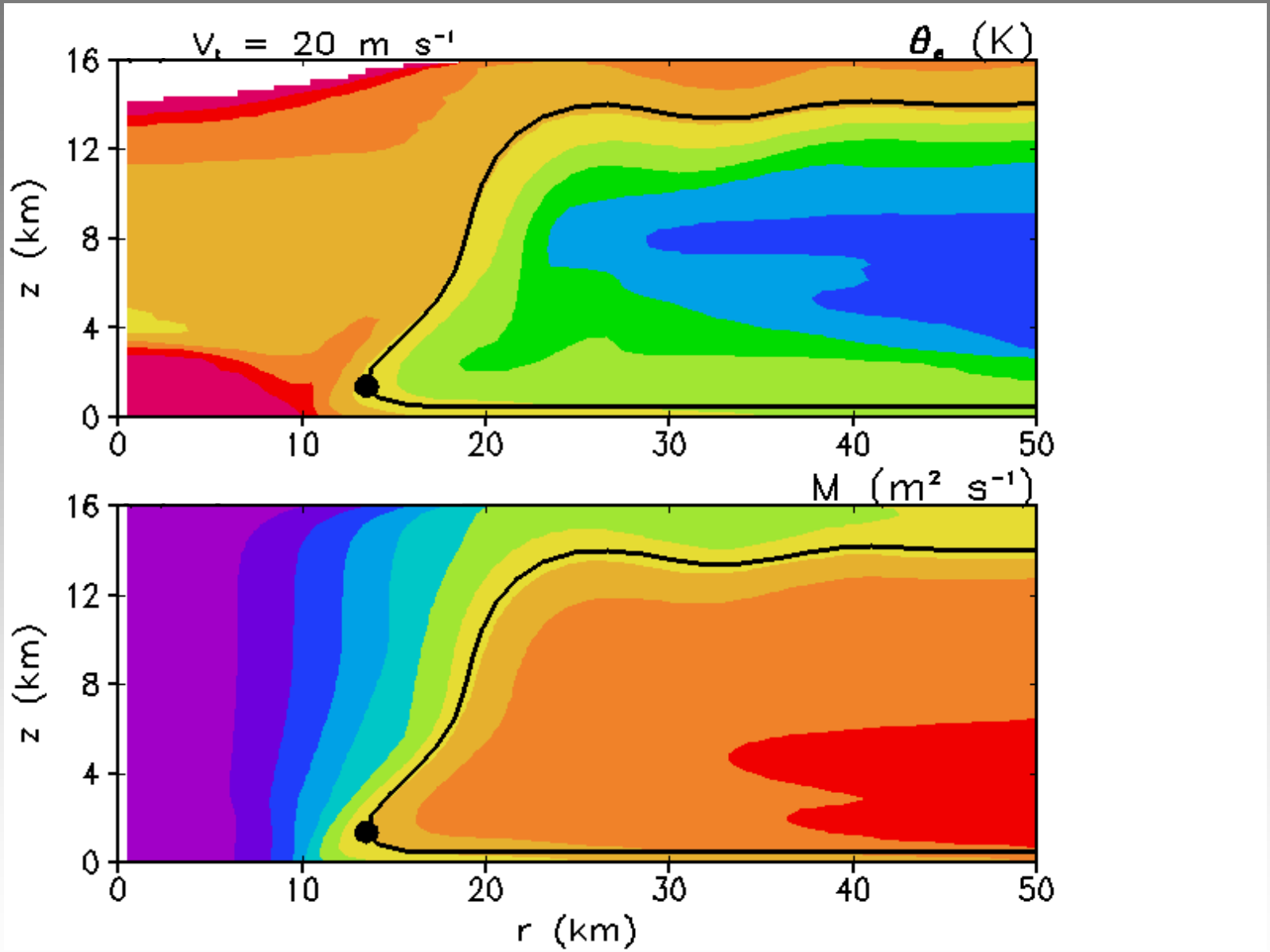
1. Moist slantwise neutrality $\boxed{s = s(M)}$

2. PBL Model

3. Gradient-wind and hydrostatic balance

Bryan and Rotunno (2009 *JAS*)

Structure of θ_e and Angular Momentum M $(l_h = 750m)$



Bryan and Rotunno (2009 *MWR*)

Components of E-PI

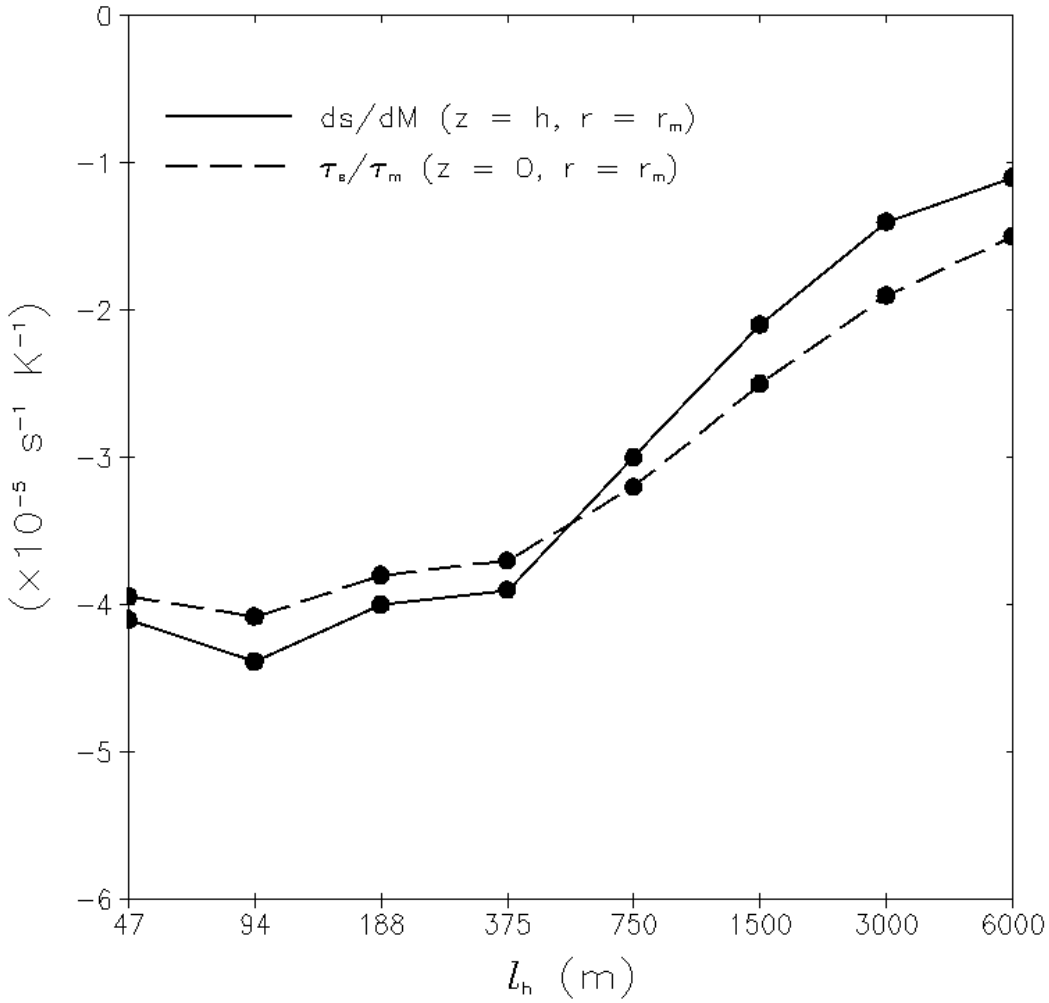
1. Moist slantwise neutrality ✓

2. PBL Model $$\boxed{\left[\frac{ds}{dM}\right]_{\substack{z=h\\r=r_m}} = -\left[\frac{\tau_s}{\tau_M}\right]_{\substack{z=0\\r=r_m}}}$$

3. Gradient-wind and hydrostatic balance

Bryan and Rotunno (2009 *JAS*)

$$\left[\frac{ds}{dM}\right]_{\substack{z=h\\r=r_m}} \stackrel{?}{=} -\left[\frac{\tau_s}{\tau_M}\right]_{\substack{z=0\\r=r_m}}$$



Bryan and Rotunno (2009 *JAS*)

Components of E-PI

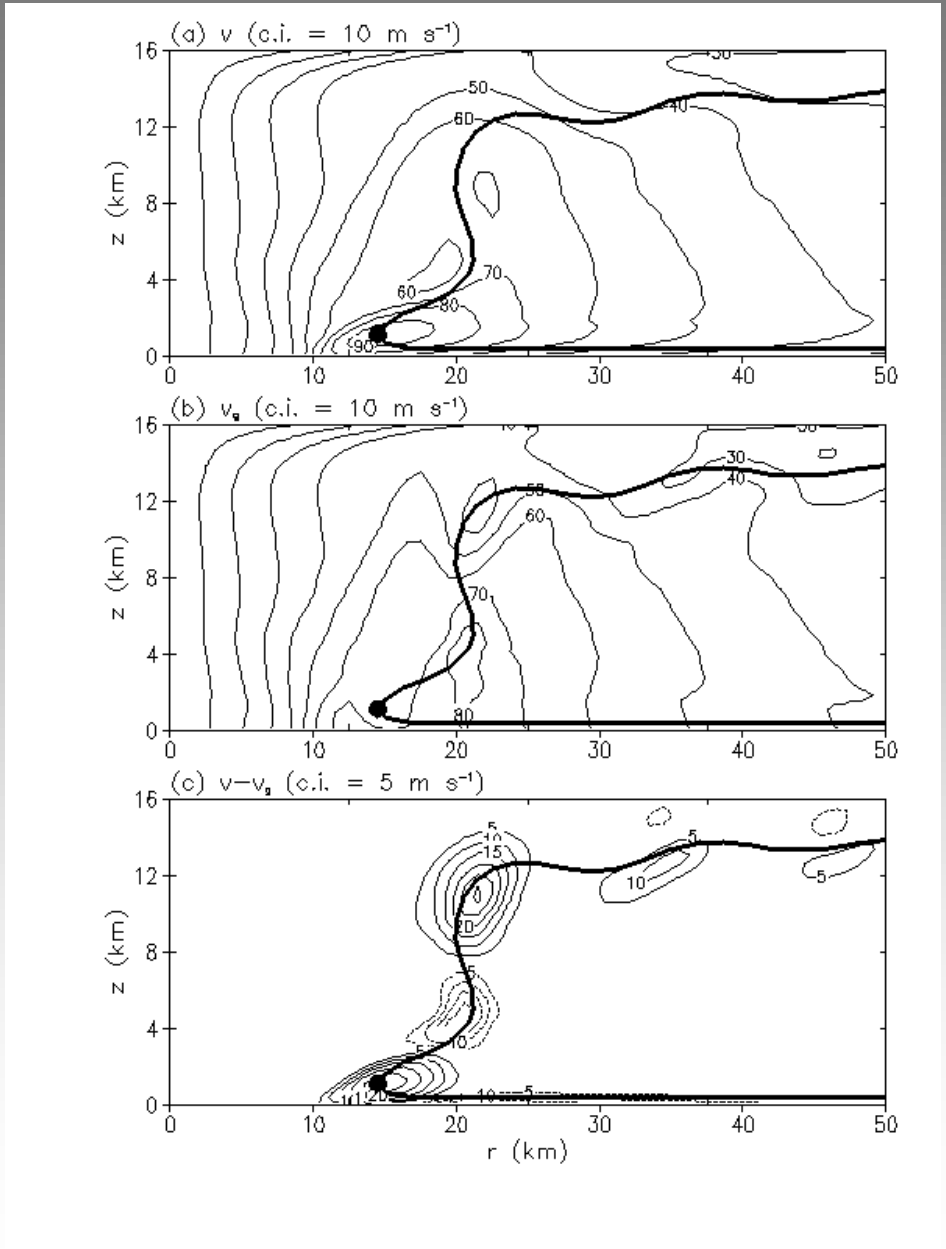
1. Moist slantwise neutrality ✓

2. PBL Model ✓

3. Gradient-wind and hydrostatic balance

Bryan and Rotunno (2009 *JAS*)

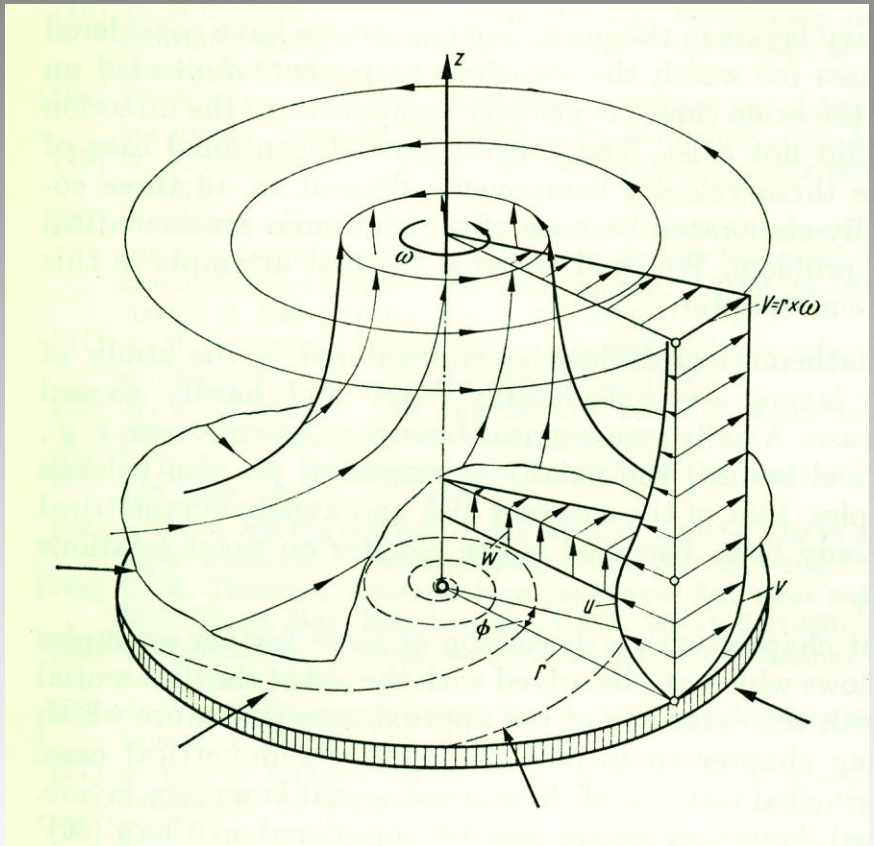
Flow with small horizontal diffusion not in gradient-wind balance

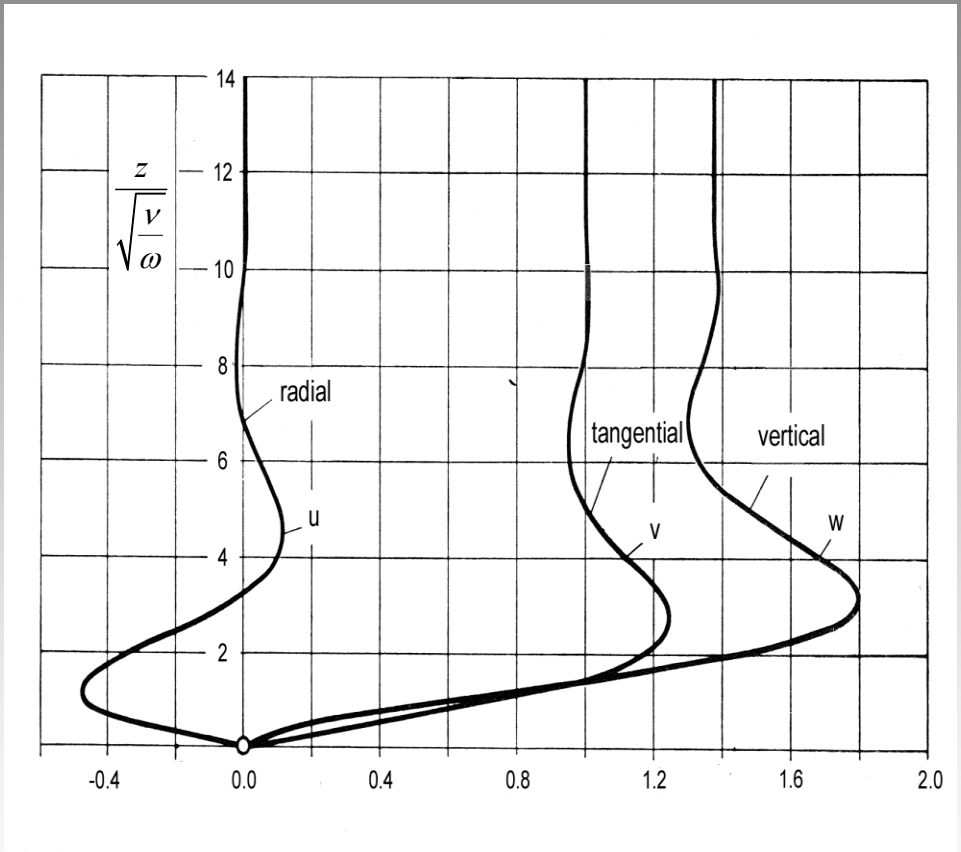


v – *azimuthal vel*
v_g – *gradient wind*
(r,z) – $(rad., alt.)$

Bryan and Rotunno (2009 *JAS*)

Rotating Flow Above a Stationary Disk*

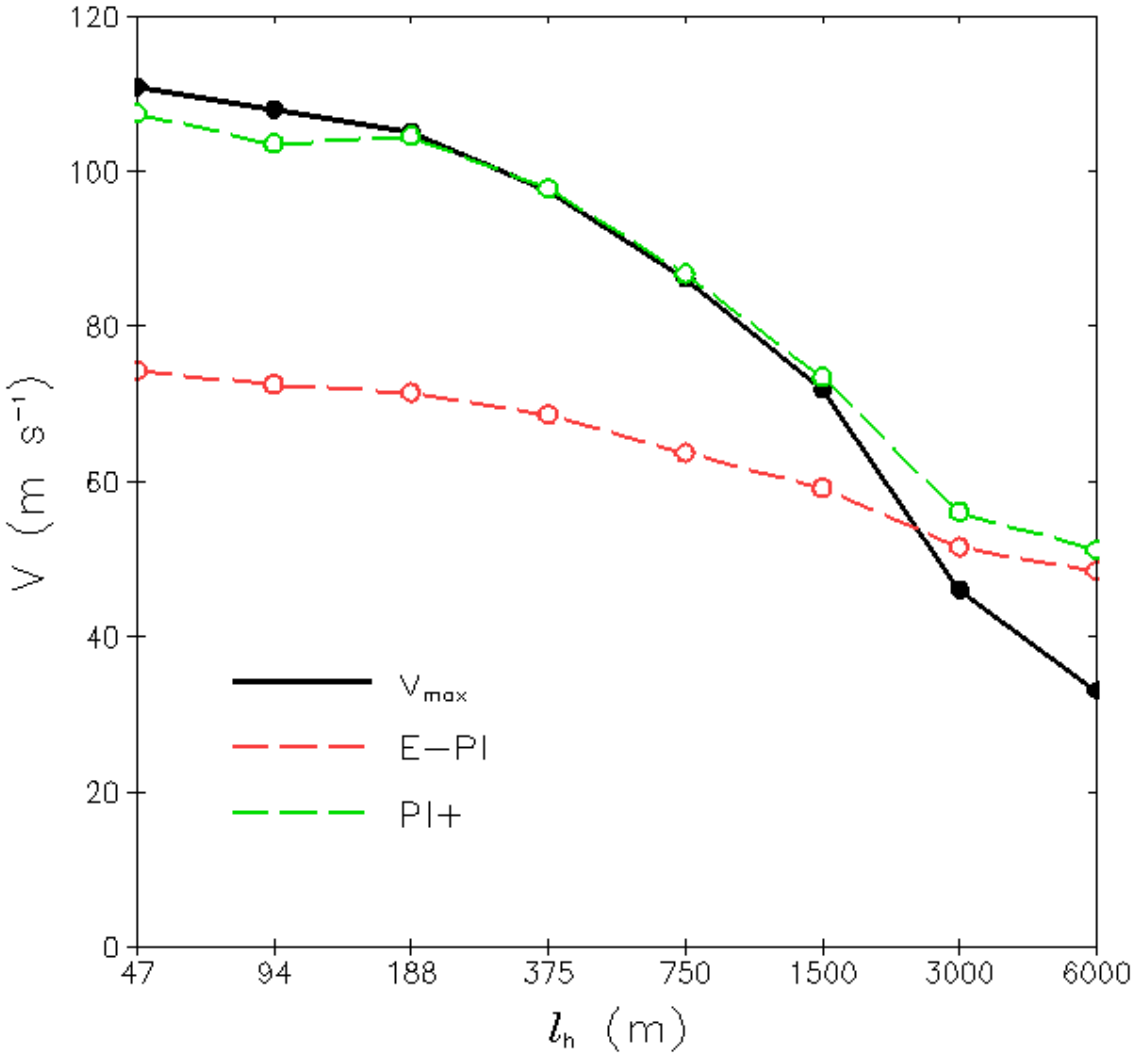




* Bödewadt (1940) (Schlichting 1968 *Boundary Layer Theory*); see also Rott and Lewellen (1966 *Prog Aero Sci)*

V_{max} , E-PI, PI$^+$ vs l_h

$$PI^+ = \sqrt{(E - PI)^2 + r_m \eta_m w_m}$$



V_{max} – max vel.

E - PI - Emanuel Pot. Intensity

l_h – hor mix.length

r_m – rad. V_{max}

η_m – azi.vort. at r_m

w_m – vert.vel. at r_m

Bryan and Rotunno (2009 *JAS*)

Components of E-PI

1. Moist slantwise neutrality ✓

2. PBL Model ✓

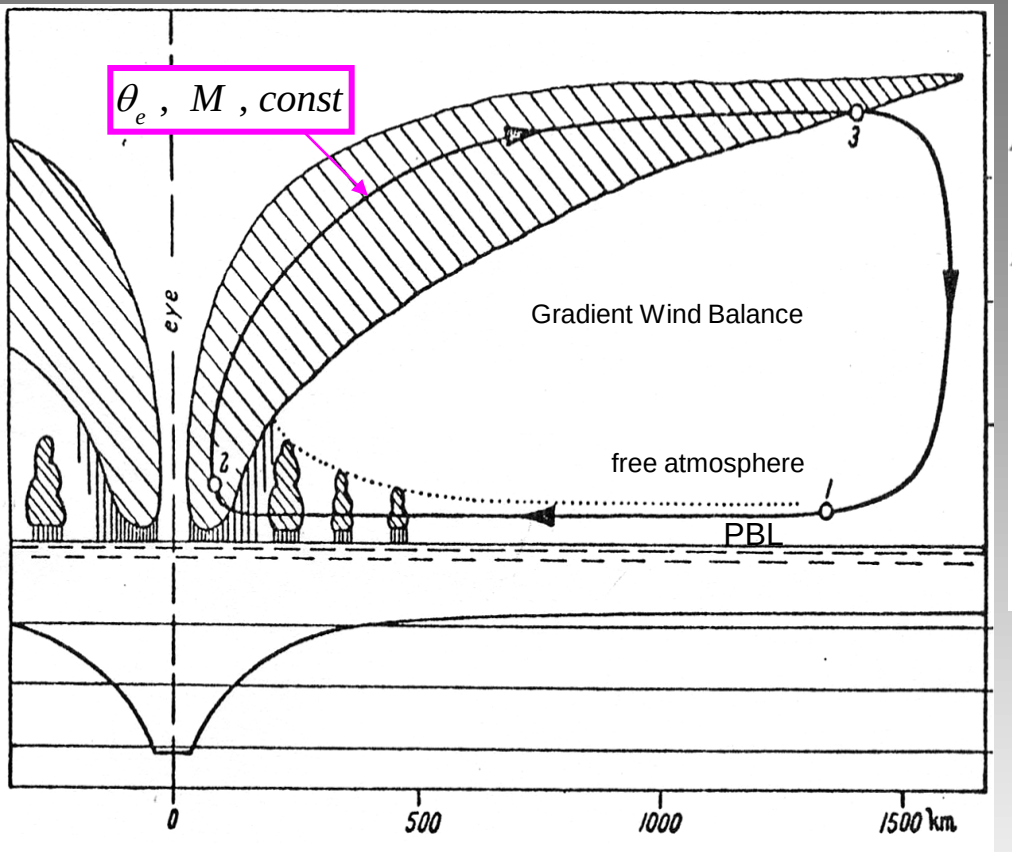
3. Gradient-wind and hydrostatic balance ✗

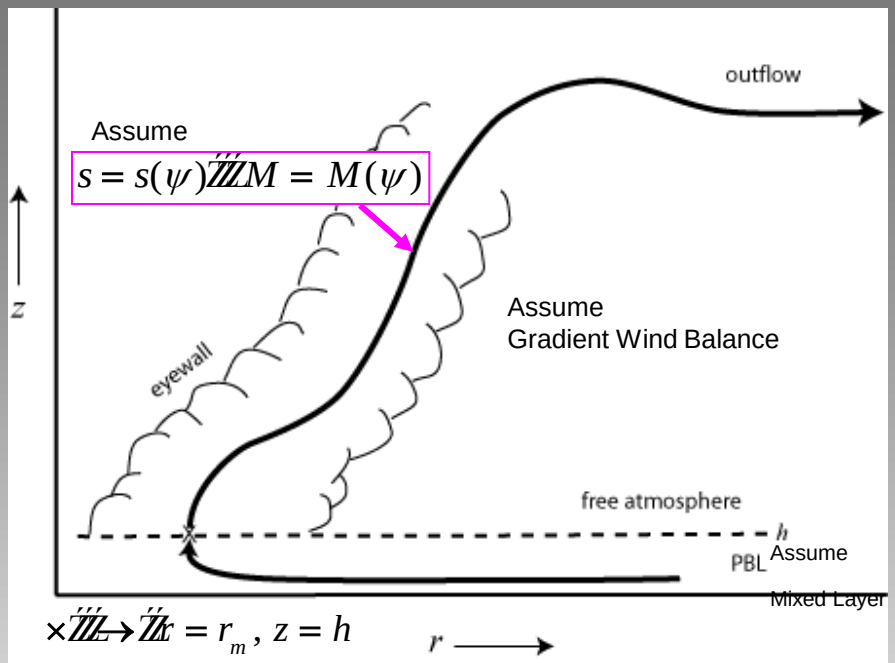
Bryan and Rotunno (2009 *JAS*)

Conclusions

- Simulated TC V_{max} > E-PI for weak horizontal diffusion
- Weak horizontal diffusion $\longrightarrow$ Violation of gradient wind
- Horizontal diffusion & non-gradient wind in natural TCs not generally known

Postscript: E-PI





Eliassen & Kleinschmidt (1957):

"From these conditions it is probably possible to explain the essential features of the hurricane…This problem, however, has not been solved."

Emanuel (1986) solved it.

In the *r-z* plane and above PBL ($\boldsymbol{F} = 0$):

$$(\boldsymbol{\omega}_a \times \boldsymbol{u} = -\frac{1}{2}\nabla q^2 - \rho^{-1}\nabla p - g\nabla z) \bullet d\boldsymbol{x} \qquad d\boldsymbol{x} = dr\,\boldsymbol{e}_r + dz\,\boldsymbol{e}_z$$

q_v – *vapor mix.rat.*
L_0 – *latent heat*
c_p – *spec.heat ct. pres.*
$\boldsymbol{u} = (u,v,w)$ – *velocity*
$\omega_{\boldsymbol{a}} = (\xi,\eta,\zeta)$ – *vorticity*
p – *pressure*
ρ – *density*
q – *speed*
g – *gravity*
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s – *entropy*
(r,z) – *(rad., alt.)*

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$$\eta\left(wdr - u\,dz\right) - v\left(\zeta dr - \xi dz\right) = -\frac{1}{2}dq^2 - \rho^{-1}dp - gdz$$

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Axisymmetry: $\rho u = r^{-1}\dfrac{\partial\psi}{\partial z}$; $\rho w = -r^{-1}\dfrac{\partial\psi}{\partial r}$; $\xi = -r^{-1}\dfrac{\partial M}{\partial z}$; $\zeta = r^{-1}\dfrac{\partial M}{\partial r}$

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Psuedo-Adiabatic Thermodynamics (Bryan 2008): $-\rho_d^{-1}dp = Tds - c_p dT - L_0 dq_v$

q_v – *vapor mix.rat.*
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Psuedo-Adiabatic Thermodynamics (Bryan 2008): $-\rho_d^{-1}dp = Tds - c_p dT - L_0 dq_v$

$$\boxed{-\frac{\eta}{\rho_d r} = \frac{1}{2r^2}\frac{dM^2}{d\psi} + T\frac{ds}{d\psi} - \frac{dE}{d\psi} - \frac{f}{2}\frac{dM}{d\psi}}$$

$$E = \frac{q^2}{2} + gz + c_p T + L_0 q_v$$

q_v – vapor mix.rat.
L_0 – latent heat
c_p – spec.heat ct. pres.
$\boldsymbol{u} = (u,v,w)$ – velocity
$\omega_a = (\xi,\eta,\zeta)$ – vorticity
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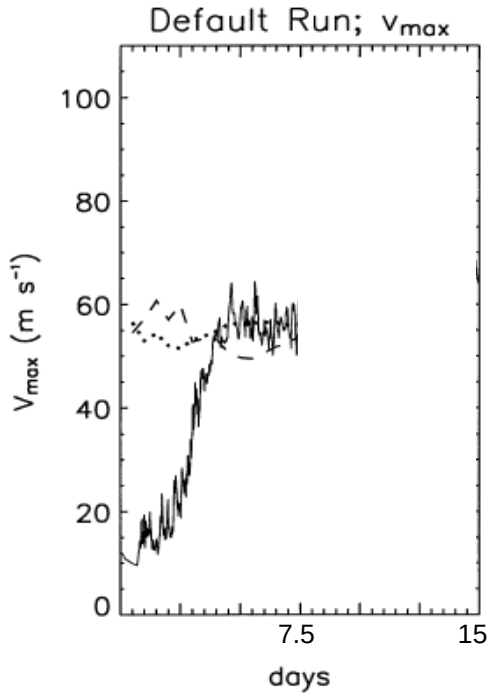

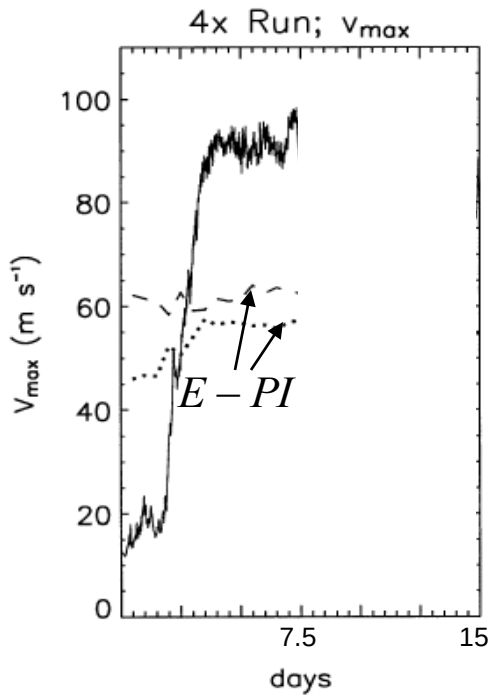

Persing and Montgomery (2003)

$\longrightarrow$

Rotunno and Emanuel (1987) Model

$V_{\max}$ Sensitive to Grid Resolution

$V_{\max} > E - PI$ **E**manuel **P**otential **I**ntensity

Bryan and Rotunno (2009 *MWR):*

RE87 paper $\longrightarrow$ $l_h = 3000m$

RE87 code $\longrightarrow$ $l_h = 0.2 \times \Delta_h$

$V_{\max}$ Sensitive to l_h

$V_{\max}$ – *max vel.*
l_h – *hor mix.length*
Δ_h – *hor.grid length*