National Weather Service
National Centers
for
Environmental Prediction

HWRF Dynamics
The WRF-NMM

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NMM Dynamic Solver

- Basic Principles
- Equations / Variables
- Model Integration
- Horizontal Grid
- Spatial Discretization
- Vertical Grid
- Boundary Conditions
- Dissipative Processes
- Namelist switches
- Summary

Basic Principles

- Use full compressible equations split into hydrostatic and nonhydrostatic contributions
    - Easy comparison of hydro and nonhydro solutions
    - Reduced computational effort at lower resolutions
- Apply modeling principles proven in previous NWP and regional climate applications
- Use methods that minimize the generation of small-scale noise
- Robust, computationally efficient

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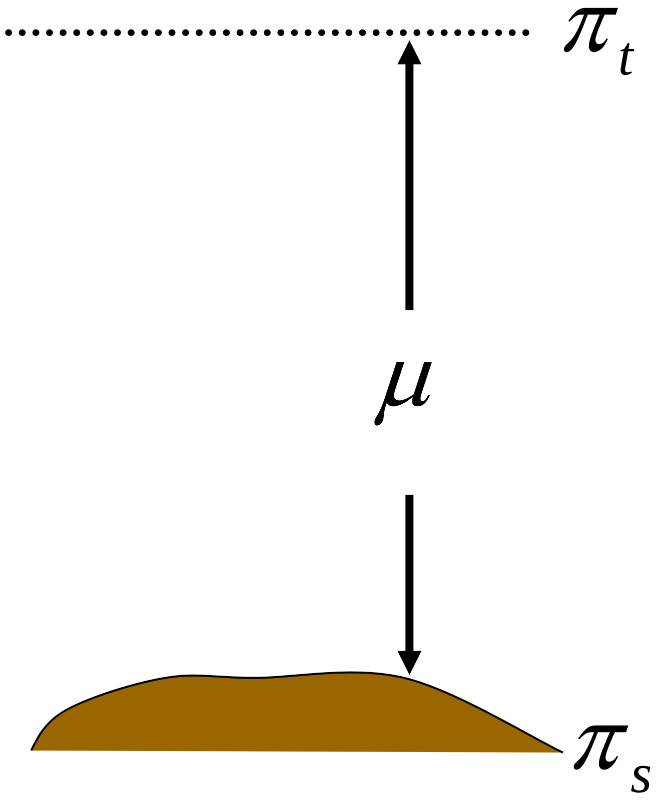
Mass Based Vertical Coordinate

To simplify discussion of the model equations, consider a sigma coordinate to represent a vertical coordinate based on hydrostatic pressure (π):

$$\mu = \pi_s - \pi_t$$

$$\sigma = \frac{\pi - \pi_t}{\mu}$$



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WRF-NMM dynamical equations

inviscid, adiabatic, sigma form

Analogous to a hydrostatic system, except for *p* and ε, where *p* is the total (nonhydrostatic) pressure and ε is defined below.

Momentum eqn.

$$\frac{\partial \mathbf{v}}{\partial t} = -\mathbf{v}\cdot\nabla_\sigma \mathbf{v} - \dot{\sigma}\frac{\partial \mathbf{v}}{\partial \sigma} - (1+\varepsilon)\nabla_\sigma \Phi - \alpha\nabla_\sigma p + f\,\mathbf{k}\times\mathbf{v}$$

Thermodynamic eqn.

$$\frac{\partial T}{\partial t} = -\mathbf{v}\cdot\nabla_\sigma T - \dot{\sigma}\frac{\partial T}{\partial \sigma} + \frac{\alpha}{c_p}\left[\frac{\partial p}{\partial t} + \mathbf{v}\cdot\nabla_\sigma p + \dot{\sigma}\frac{\partial p}{\partial \sigma}\right]$$

Hydrostatic Continuity eqn.

$$\frac{\partial \mu}{\partial t} + \nabla_\sigma\cdot(\mu\mathbf{v}) + \frac{\partial(\mu\dot{\sigma})}{\partial \sigma} = 0$$

$$\boxed{\varepsilon \equiv \frac{1}{g}\frac{dw}{dt}}$$

$$\boxed{\alpha = RT/p}$$

Janjic et al. 2001, *MWR*

*

Hypsometric eqn.

$$\frac{\partial \Phi}{\partial \sigma} = -\mu \frac{RT}{p}$$

Nonhydro var. definition (restated)

$$\varepsilon \equiv \frac{1}{g}\frac{dw}{dt}$$

3rd eqn of motion

$$\frac{\partial p}{\partial \pi} = 1 + \varepsilon$$

ε generally is small. Even a large vertical acceleration of 20 m/s in 1000 s produces ε of only ~0.002, and nonhydrostatic pressure deviations of ~200 Pa.

Nonhydrostatic continuity eqn.

$$w = \frac{1}{g}\frac{d\Phi}{dt} = \frac{1}{g}\left(\frac{\partial \Phi}{\partial t} + \mathbf{v} \bullet \nabla_{\sigma}\Phi + \dot{\sigma}\frac{\partial \Phi}{\partial \sigma}\right)$$

*

Properties of system

- Φ, *w*, and ε are not independent → no independent prognostic equation for *w*!
- $\varepsilon \ll 1$ in meso- and large-scale atmospheric flows.
- Generically, the impact of nonhydrostatic dynamics becomes detectable at resolutions < 10 km, and important at ~1 km.

Vertical boundary conditions for model equations*

Top: $\dot{\sigma} = 0$ **,** $p - \pi = 0$

Surface: $\dot{\sigma} = 0$ **,** $\dfrac{\partial (p - \pi)}{\partial \sigma} = 0$

WRF-NMM predictive variables

- Mass variables:
    - **`PD`** – hydrostatic pressure depth (time/space varying component) (Pa)
    - **`PINT`** – nonhydrostatic pressure (Pa)
    - **`T`** – sensible temperature (K)
    - **`Q`** – specific humidity (kg/kg)
    - **`CWM`** – total cloud water condensate (kg/kg)
    - **`Q2`** – 2 * turbulent kinetic energy (m^2/s^2)
- Wind variables:
    - **`U,V`** – wind components (m/s)

Model Integration

- Explicit time differencing preferred where possible, as allows for better phase speeds and more transparent coding:
  - horizontal advection of u, v, T
  - advection of q, cloud water, TKE (“passive substances”)

- Implicit time differencing for very fast processes that would require a restrictively short time step for numerical stability:
  - vertical advection of u, v, T and vertically propagating sound waves

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Model Integration

Horizontal advection of u, v, T

2nd order Adams-Bashforth:

$$\frac{y^{\tau+1}-y^{\tau}}{\Delta t}=\frac{3}{2}f(y^{\tau})-\frac{1}{2}f(y^{\tau-1})$$

Stability/Amplification:

A-B has a weak linear instability (amplification) which either can be tolerated or can be stabilized by a slight off-centering as is done in the WRF-NMM.

$$\frac{y^{\tau+1}-y^{\tau}}{\Delta t}=1.533f(y^{\tau})-0.533f(y^{\tau-1})$$

*

Model Integration

Vertical advection of u, v, & T

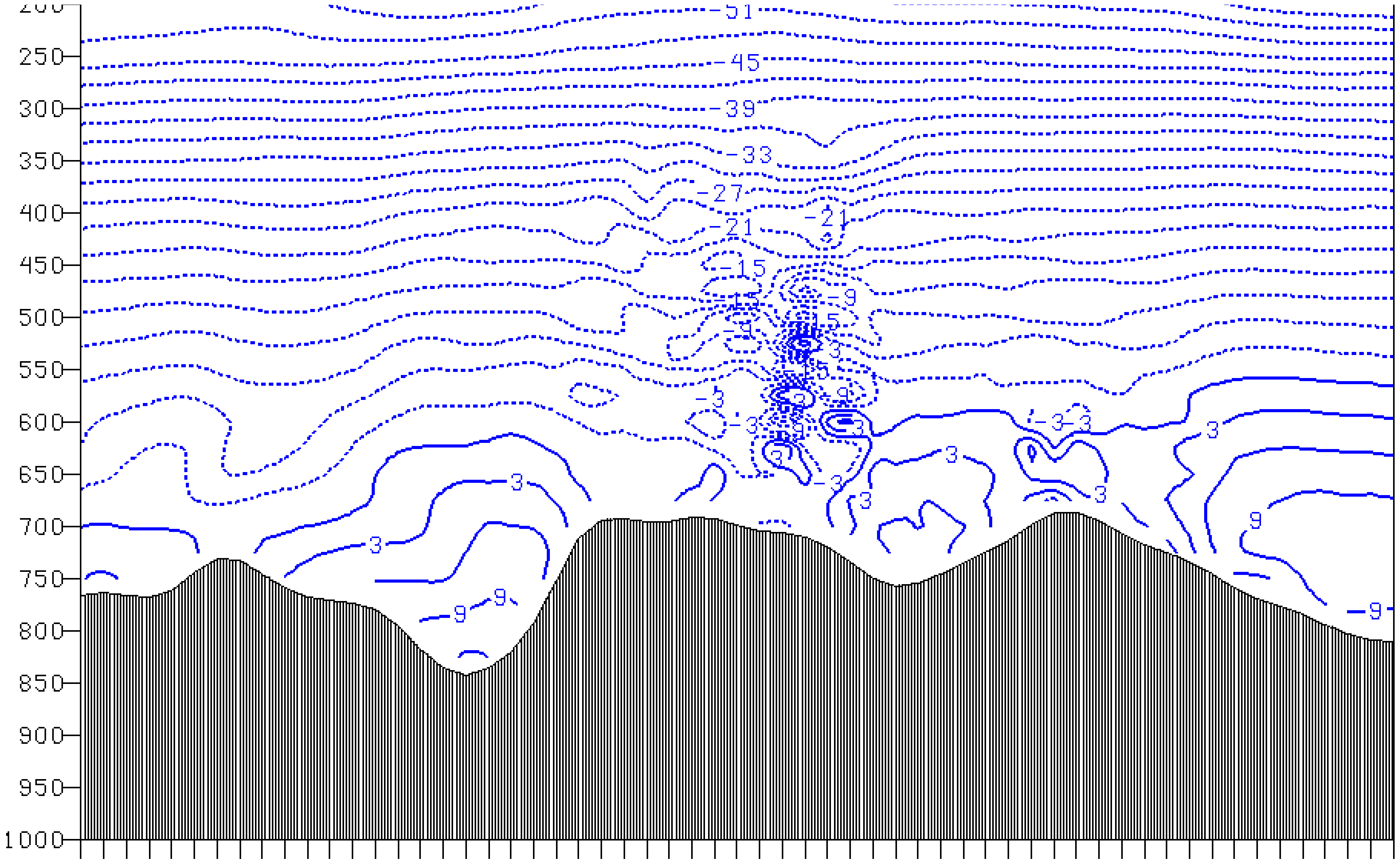
Crank-Nicolson (w/ off centering in time):

$$\frac{y^{\tau+1} - y^{\tau}}{\Delta t} = \frac{1}{2}[1.1 f(y^{\tau+1}) + 0.9 f(y^{\tau})]$$

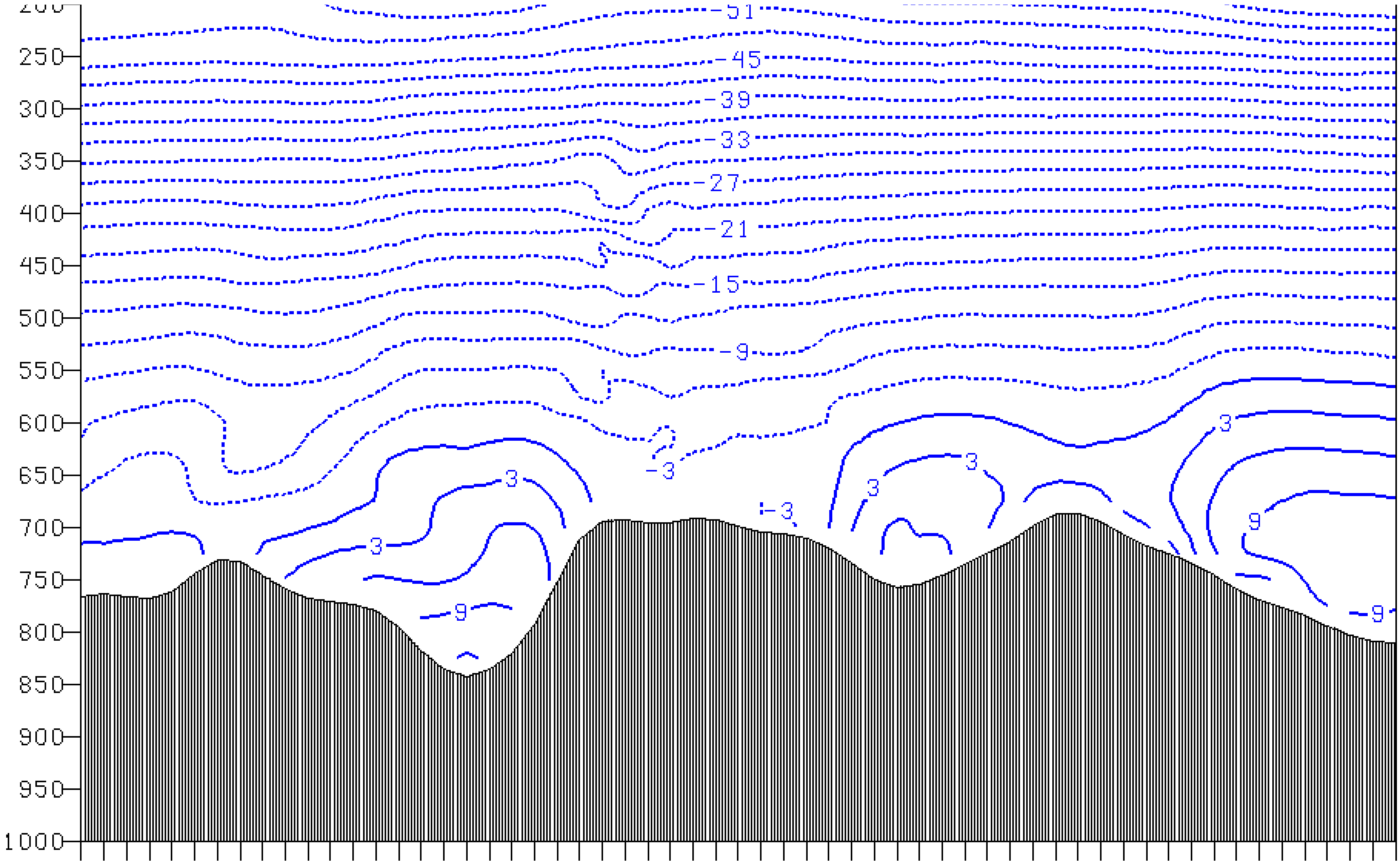
Stability:

An implicit method, it is absolutely stable numerically. Short time steps still needed for *accuracy.*

Cross-section of temperatures 18 h into an integration experiencing strong orographically-forced vertical motion and using **centered in time** C-N vertical advection



Cross-section of temperatures 18 h into an integration experiencing strong orographically-forced vertical motion and using **off-centered in time** C-N vertical advection.



Model Integration

Advection of TKE (Q2) and moisture (Q, CWM, species)

- Traditionally has taken an approach similar to the Janjic (1997) scheme used in Eta model:
  - Starts with an initial upstream advection step
  - Anti-diffusion/anti-filtering step applied to reduce dispersiveness
  - Conservation enforced after each anti-filtering step
    - maintain global sum of advected quantity
    - prevent generation of new extrema
- Eulerian advection also available, but not used by HWRF.

Model Integration

- Subroutine sequence within solve_nmm (ignoring physics):
  - (3%) PDTE – integrates mass flux divergence, computes vertical velocity and updates hydrostatic pressure.
  - (21%) ADVE – horizontal and vertical advection of T, u, v, Coriolis and curvature terms applied.
  - (32%) ADV2/ADV2_SCAL (typically every other step) – vertical/horizontal advection of q, CWM, TKE
  - (1%) VTOA – updates nonhydrostatic pressure, applies $\omega\alpha$ term to thermodynamic equation
  - (6%) VADZ/HADZ – vertical/horizontal advection of height. w=dz/dt updated.

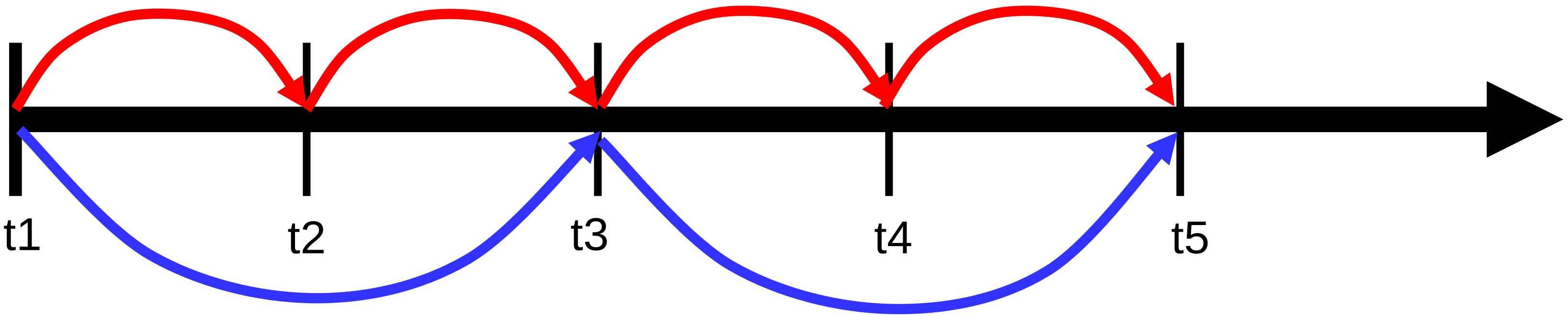
(*approximate* relative % of dynamics time spent in these subroutines)

Model Integration

- Subroutine sequence within solve_nmm (cont):
  - (9%) EPS – vertical and horizontal advection of dz/dt, vertical sound wave treatment.
  - (11%) HDIFF – horizontal diffusion
  - (<1%) BOCOH – boundary update at mass points
  - (14%) PFDHT – calculates PGF, updates winds due to PGF, computes divergence.
  - (1%) DDAMP – divergence damping
  - (<1%) BOCOV – boundary update at wind points\

  - Nest motion
  - Diagnostics

All dynamical processes every fundamental time step, except….



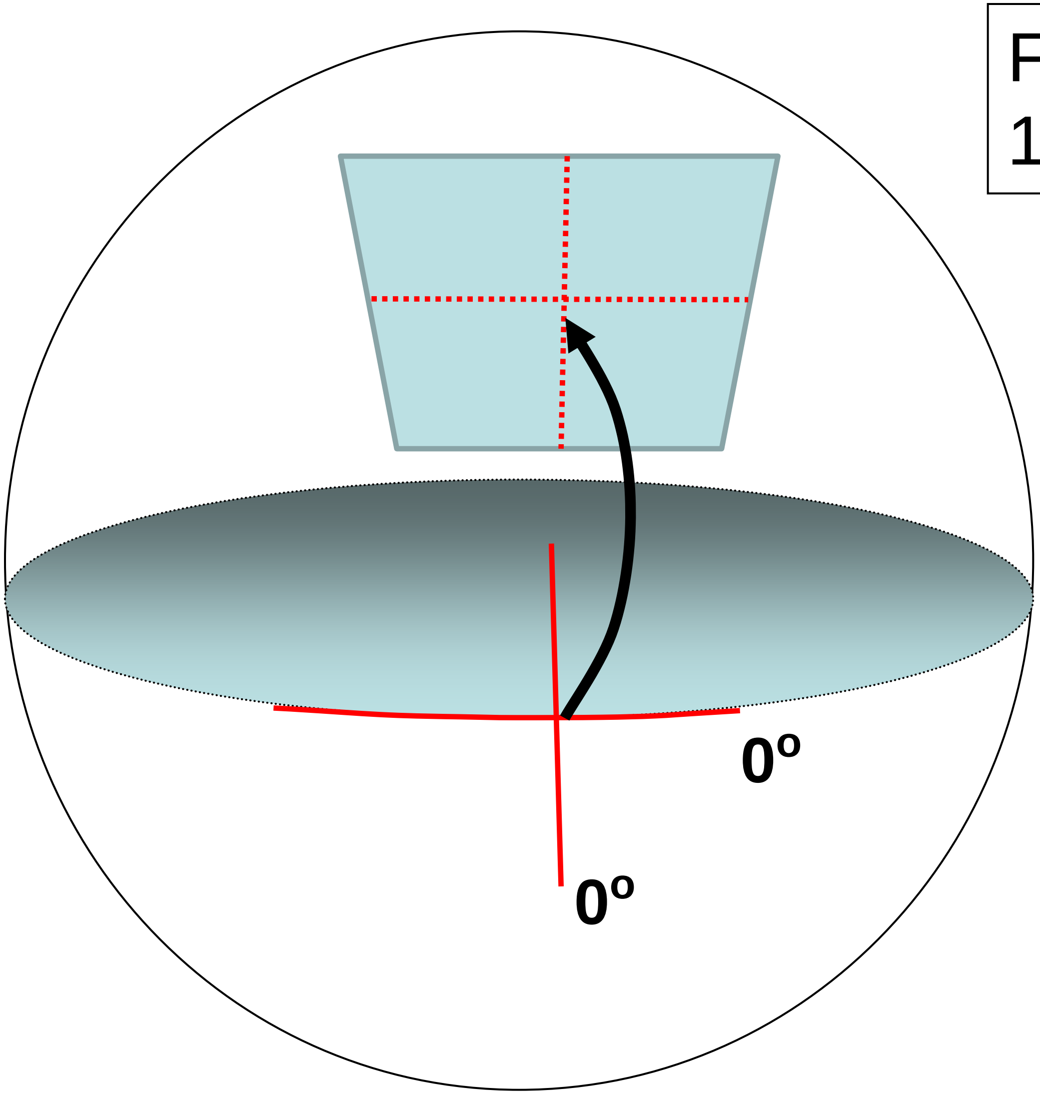
…passive substance advection, every other time step

Model time step "dt" specified in model namelist.input is for the fundamental time step.

Generally about 2.25x** the horizontal grid spacing (km), or 350x the namelist.input "dy" value (degrees lat).

** runs w/o parameterized convection may benefit from limiting the time step to about 1.9-2.0x the grid spacing.

Impact on variation of Δx over domain



For a domain spanning
10N to 70N:

$$\Delta x \propto \cos(lat)$$

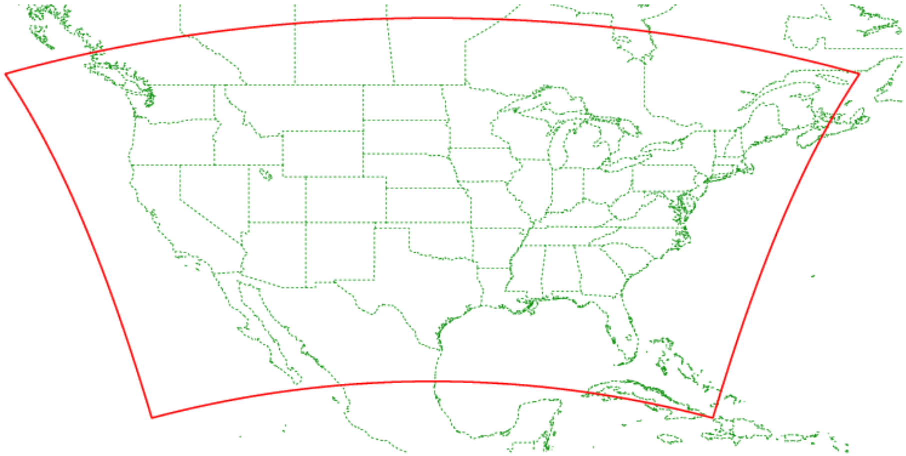
Regular lat-lon grid

$$\cos(70^0)/\cos(10^0) = 0.347$$

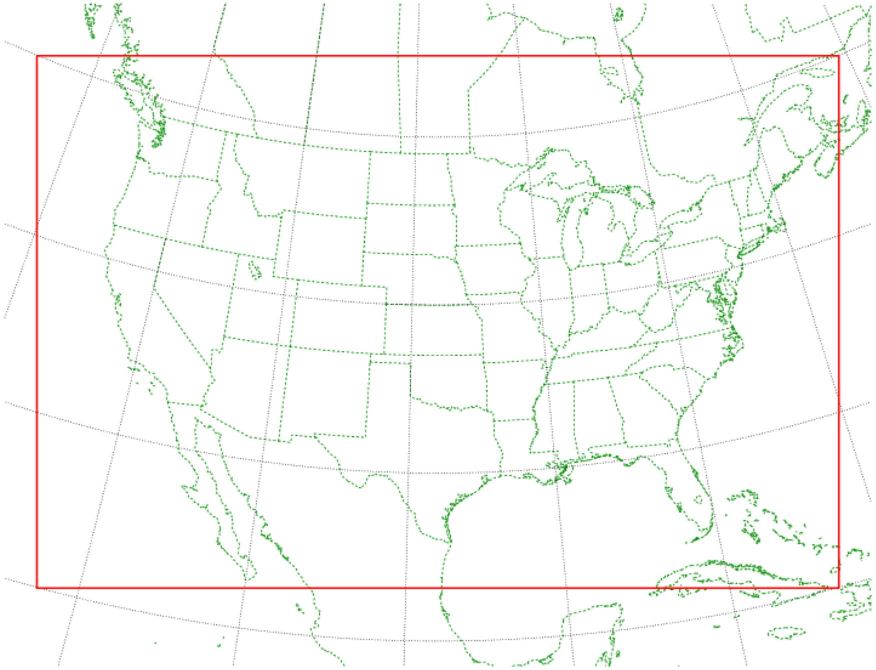
Rotated lat-lon grid

$$\cos(30^0)/\cos(0^0) = 0.866$$

Sample rotated lat-lon domain

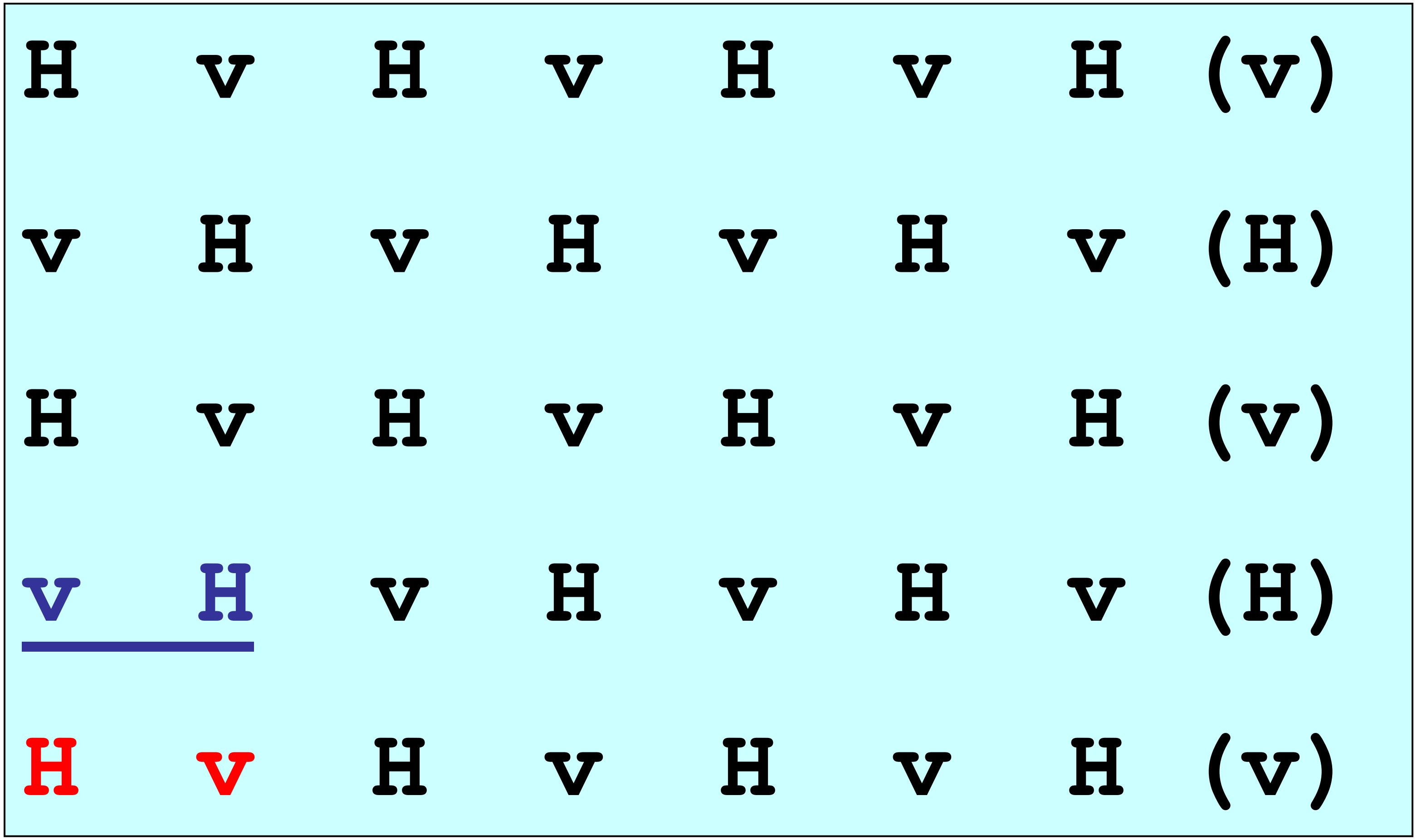


On a regular lat-lon map background



On a rotated lat-lon map background (same rotation as model grid).

The E-grid Stagger

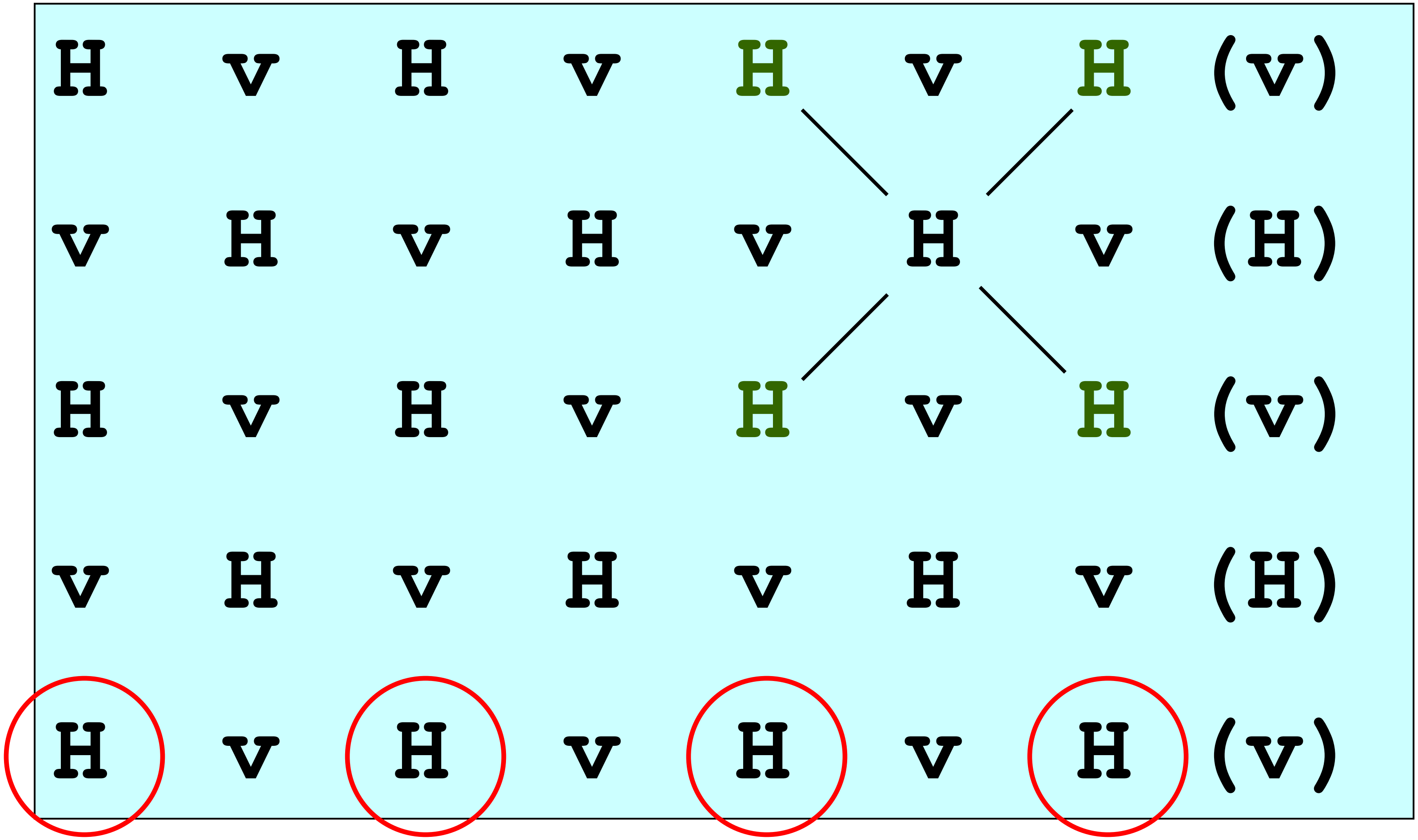


H	**v**	**H**	**v**	**H**	**v**	**H**	**(v)**
v	**H**	**v**	**H**	**v**	**H**	**v**	**(H)**
H	**v**	**H**	**v**	**H**	**v**	**H**	**(v)**
v	**H**	**v**	**H**	**v**	**H**	**v**	**(H)**
H	**v**	**H**	**v**	**H**	**v**	**H**	**(v)**

H=mass point, v=wind point

red=(1,1) ; blue=(1,2)

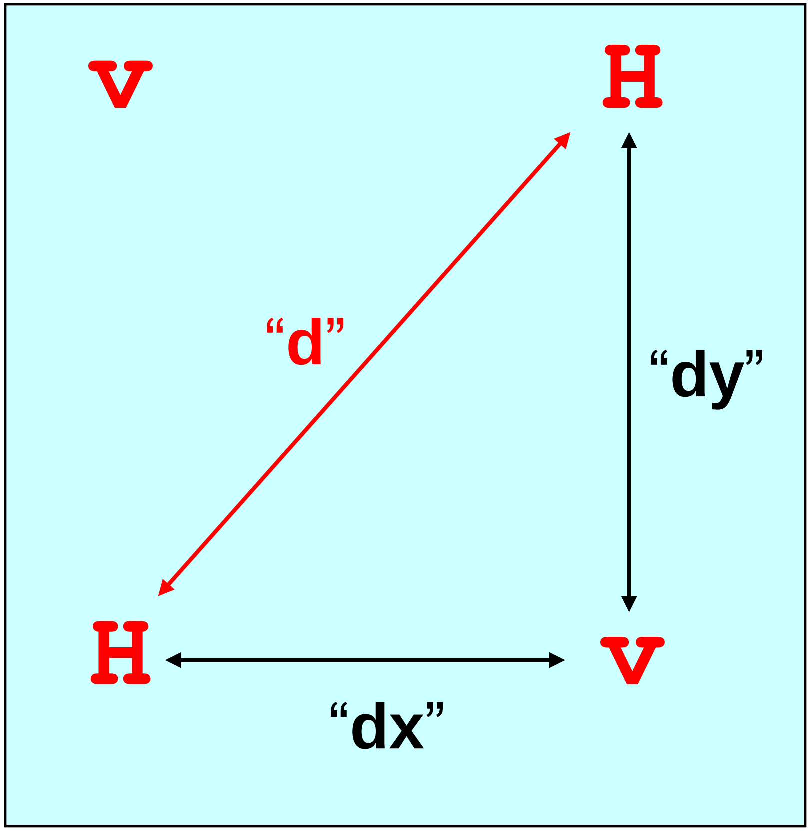
The E-grid Stagger



H	v	H	v	H	v	H	(v)
v	H	v	H	v	H	v	(H)
H	v	H	v	H	v	H	(v)
v	H	v	H	v	H	v	(H)
H	v	H	v	H	v	H	(v)

XDIM=4 (# of mass points on odd numbered row)
YDIM=5 (number of rows)

The E-grid Stagger



- Conventional grid spacing is the diagonal distance "**d**".

- Grid spacings in the WPS and WRF namelists are the "**dx**" and "**dy**" values, *specified in fractions of a degree for the WRF-NMM*.

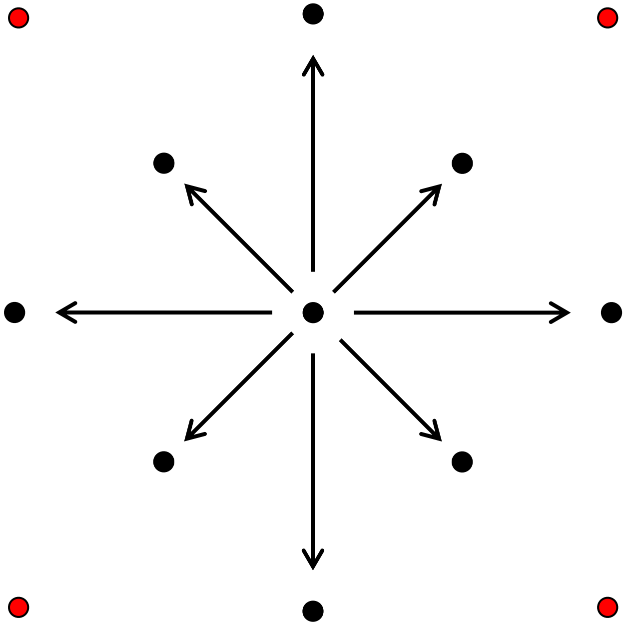
- "WRF domain wizard" takes input grid spacing "**d**" in km and computes the angular distances "**dx**" and "**dy**" for the namelist.

Spatial Discretization

General Philosophy

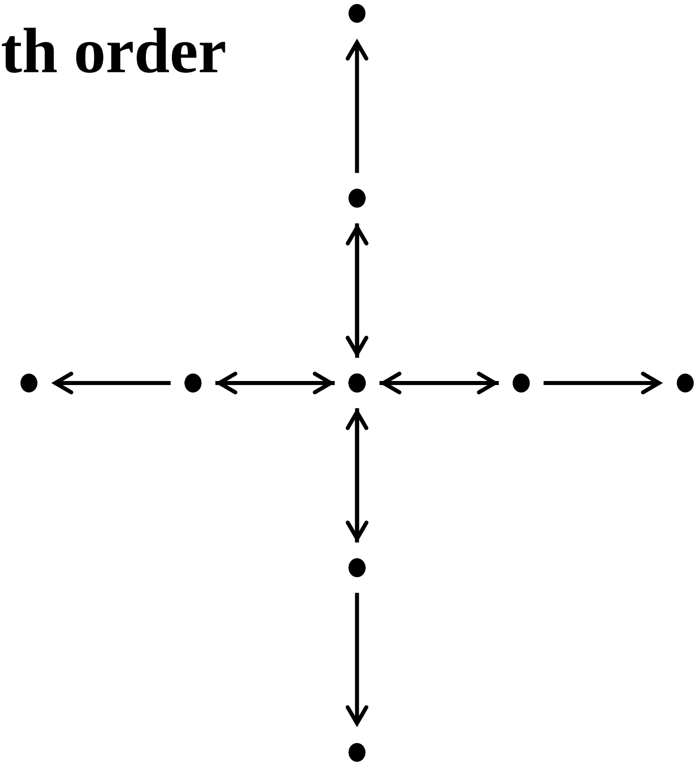
- “Mimetic” approach
  - http://www.math.unm.edu/~stanly/mimetic/mimetic.html
- Conserve energy and enstrophy to control nonlinear energy cascade
  - reduce the need for numerical filtering
- Conserve some first order and quadratic quantities
  - (mass, momentum, energy, ...)
- Use consistent order of accuracy for advection and divergence operators and the omega-alpha term; consistent transformations between KE and PE in the hydrostatic limit.
- Preserve properties of differential operators.

NMM



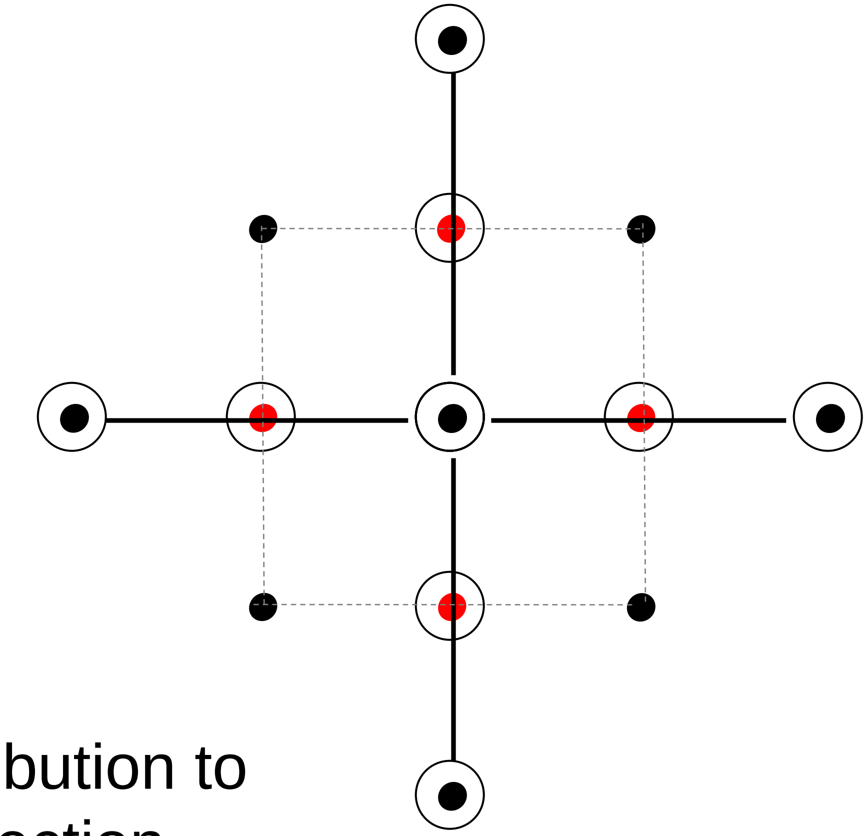
Advection and divergence operators – each point talks to all eight neighboring points (isotropic)

Formal
4th order



● mass point

🔴 wind point

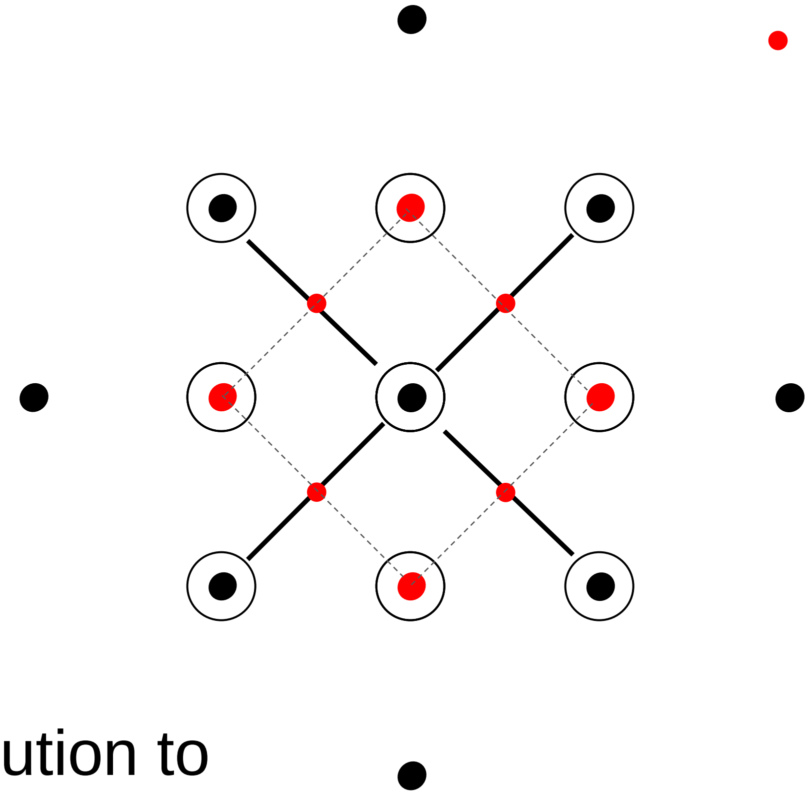


1/3 of the contribution to divergence/advection comes from these N/S and E/W fluxes.

● mass point

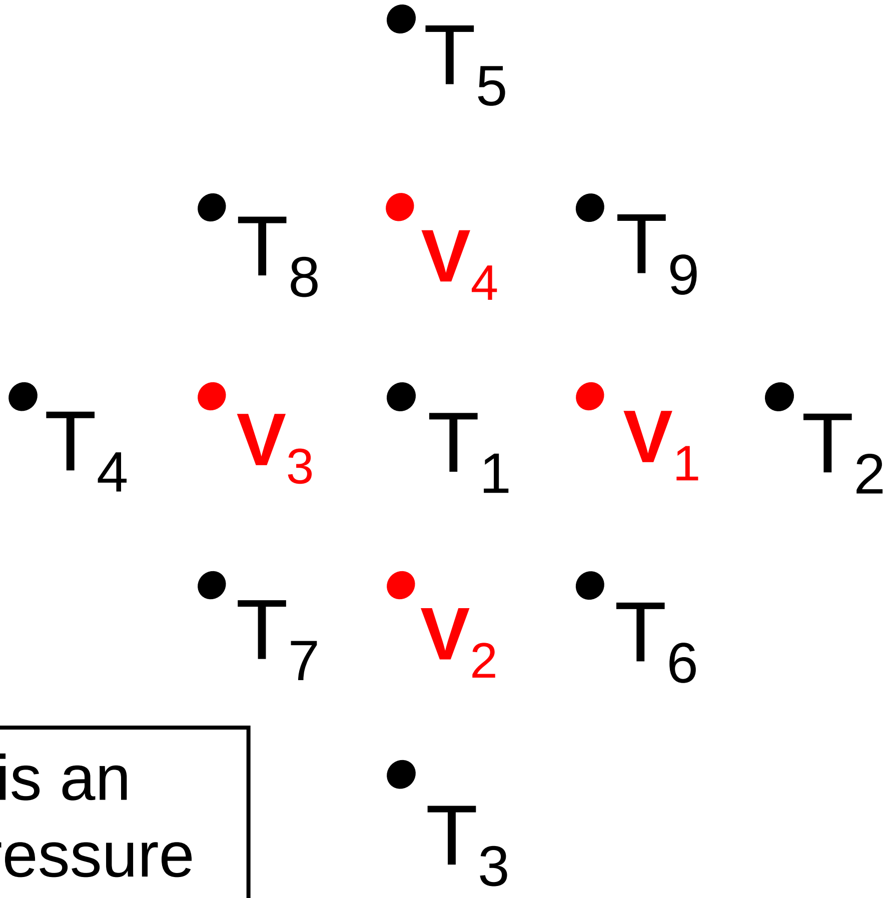
● wind point

• avg wind point



2/3 of the contribution to divergence/advection comes from these diagonal fluxes.

Horizontal temperature advection detail (computerese)



For each T_n, there is an associated layer pressure depth (here denoted dp_n). There also is a dx_n specific to each point

NMM Vertical Coordinate

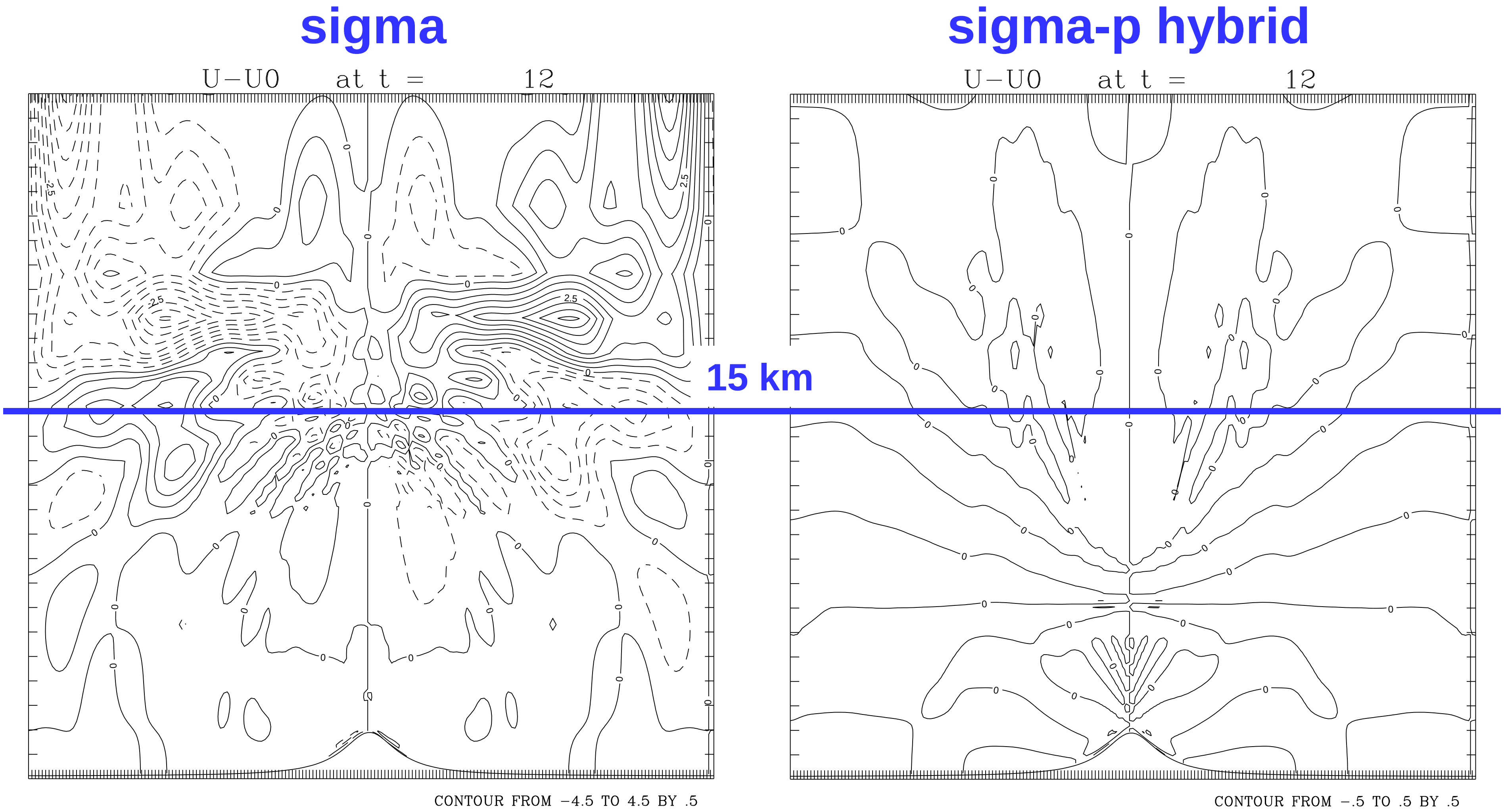
Pressure-sigma hybrid (Arakawa and Lamb, 1977)

Has the desirable properties of a terrain-following pressure coordinate:

- Exact mass (etc.) conservation
- Nondivergent flow remains on pressure surfaces
- No problems with weak static stability
- No discontinuities or internal boundary conditions

And an additional benefit from the hybrid:

- Flat coordinate surfaces at high altitudes where sigma problems worst (e.g., Simmons and Burridge, 1981)



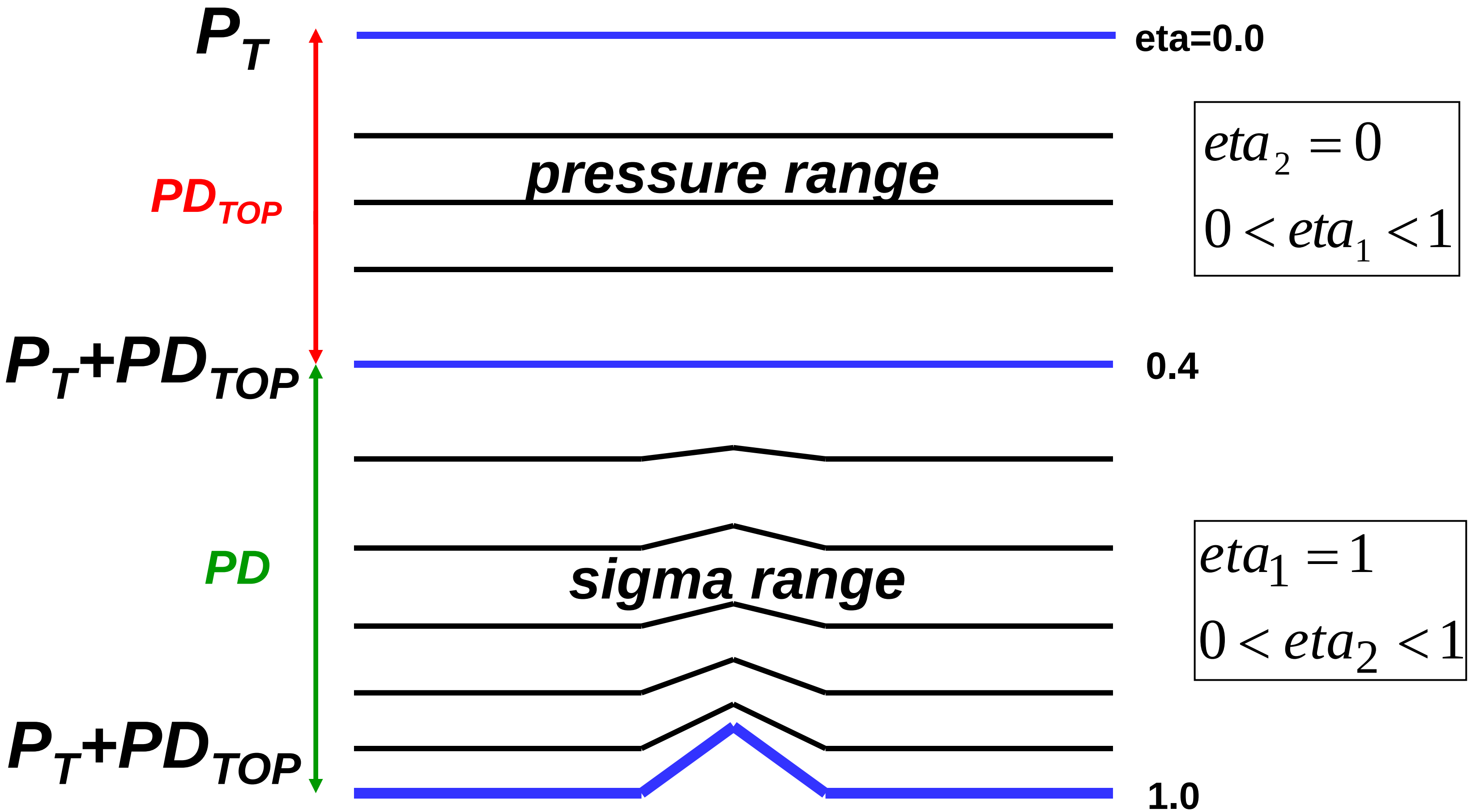
sigma

sigma-p hybrid

15 km

Wind developing due to the spurious pressure gradient force in an idealized integration. The hybrid coordinate boundary between the pressure and sigma domains is at ~400 hPa.

Pressure-Sigma Hybrid Vertical Coordinate



P_T eta=0.0

PD_{TOP}

pressure range

$$eta_2 = 0$$
$$0 < eta_1 < 1$$

$P_T + PD_{TOP}$ 0.4

PD

sigma range

$$eta_1 = 1$$
$$0 < eta_2 < 1$$

$P_T + PD_{TOP} + PD$ 1.0

$$\pi = eta_1 * PD_{TOP} + eta_2 * PD + P_T$$

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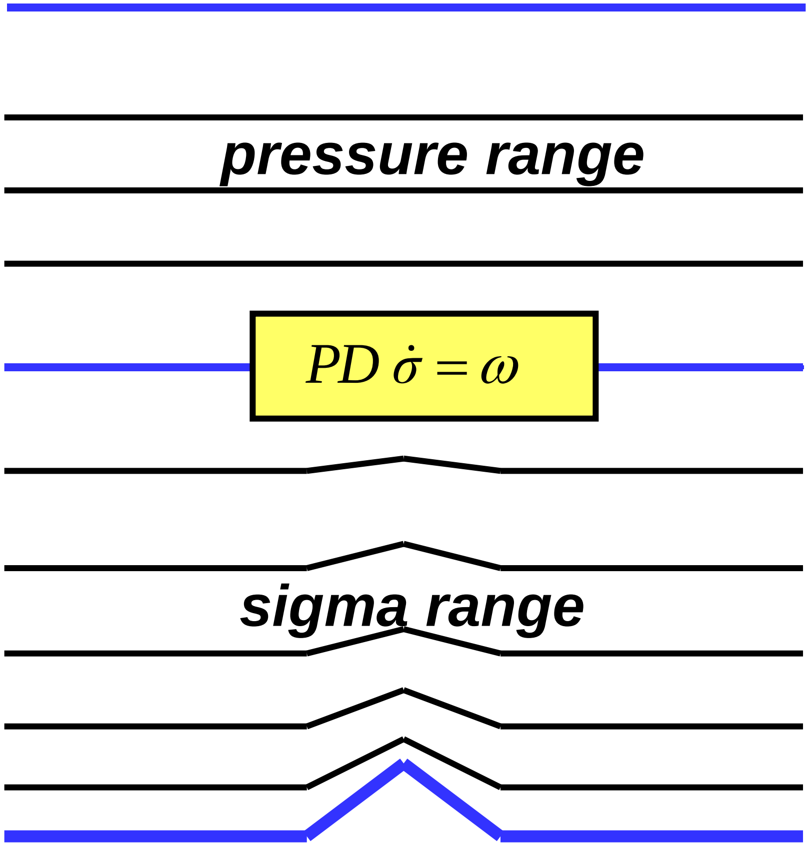
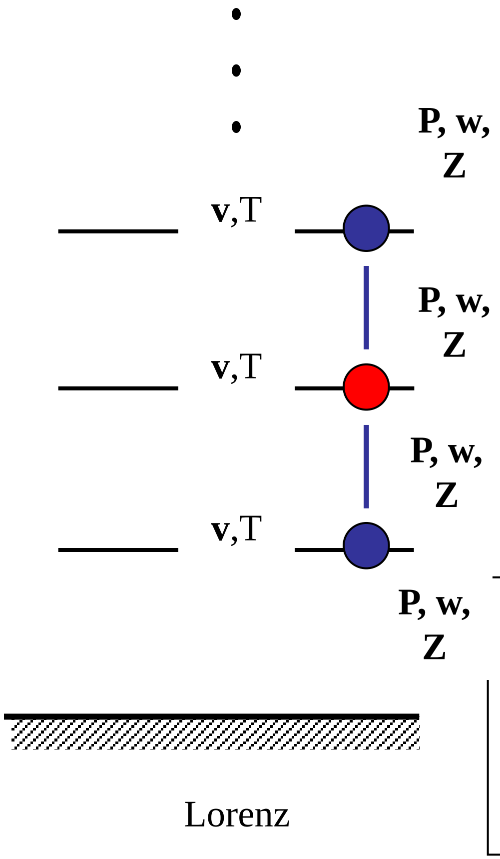
Equations in Hybrid Coordinate



$$\nabla_p \bullet (\mathbf{v}) + \frac{\partial \omega}{\partial p} = 0$$

$$\frac{\partial PD}{\partial t} + \nabla_\sigma \bullet (PD\ \mathbf{v}) + \frac{\partial (PD\,\dot{\sigma})}{\partial \sigma} = 0$$

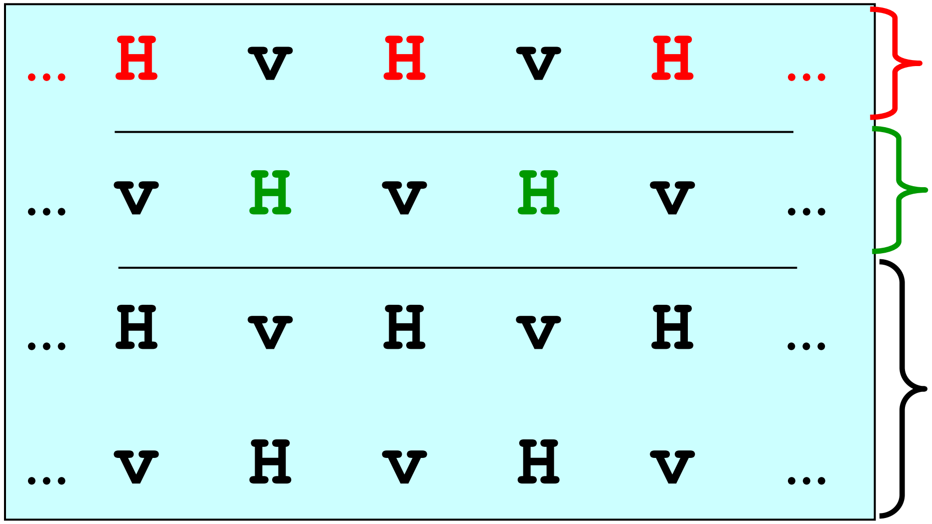
Vertical discretization



Vertical advection combines the advective fluxes computed above and below the layer of interest.

Lateral Boundary Conditions

- Lateral boundary information prescribed only on outermost row:



Pure boundary information

Avg of surrounding H points (blends boundary and interior)

Freely evolving

- Upstream advection in three rows next to the boundary
  - No computational outflow boundary condition for advection

- Enhanced divergence damping close to the boundaries.

Dissipative Processes – lateral diffusion

A 2nd order, nonlinear Smagorinsky-type horizontal diffusion is utilized:

- Diffusion strength a function of the local TKE, deformation of the 3D flow, and a namelist-specified diffusion strength variable (*coac*).
- Lateral diffusion is zeroed for model surfaces sloping more than 4.5 m per km (0.0045) by default.
- This slope limit can be adjusted with the namelist variable *slophc*. *slophc* is expressed as sqrt(2) times the true slope (making the 0.0045 default ~0.00636)

Dissipative Processes - divergence damping *

- Internal mode damping (on each vertical layer)

$$\boxed{\mathrm{v}_j = \mathrm{v}_j + \frac{(\nabla \cdot dp_{j+1}\vec{\mathbf{v}}_{j+1} - \nabla \cdot dp_{j-1}\vec{\mathbf{v}}_{j-1})}{(dp_{j+1} + dp_{j-1})} \cdot DDMPV}$$

- External mode damping (vertically integrated)

$$\boxed{\mathrm{v}_j = \mathrm{v}_j + \frac{(\int \nabla \cdot dp_{j+1}\vec{\mathbf{v}}_{j+1} - \int \nabla \cdot dp_{j-1}\vec{\mathbf{v}}_{j-1})}{(\int dp_{j+1} + \int dp_{j-1})} \cdot DDMPV}$$

$$DDMPV \approx \sqrt{2} \cdot dt \cdot CODAMP$$

CODAMP is a namelist controlled variable = 6.4 by default.

Gravity Wave Drag & Mountain Blocking

- Accounts for sub-grid scale mountain effects: mountain waves (GWD) and stability-dependent blocking of low-level flow around topography (MB).
- More important for coarser grid spacing (> ~10 km) and longer (multi-day) integrations.
- gwd_opt=2 in physics namelist to invoke for the WRF-NMM.
- Benefits overall synoptic patterns and near-surface wind and temperature forecasts.
- Based on the GFS model package for GWD (Alpert et al., 1988, 1996; Kim & Arakawa,1995) and MB (Lott & Miller, 1997).

Courtesy of Brad Ferrier

Relatively new NMM namelist switches

```
&dynamics
wp                             = 0.00
coac                           = 0.75,
codamp                         = 6.4,
slophc                         = 0.006364
```

Defaults

- **WP** - off-centering weight in nonhydrostatic computation (value of ~0.1 improves stability of some sub-1.5 km grid forecasts).
- **COAC** - diffusion strength (larger → more diffusive smoothing)
- **CODAMP** - divergence damping strength (larger → more damping, fewer small-scale regions of divergence).
- **SLOPHC** - max surface slope for diffusion (larger value applies lateral diffusion over more mountainous terrain).

A corrected namelist switch in WRFV3.4

```
&dynamics
non_hydrostatic                  = .true.
```

Default

- If .false., the model will run as a hydrostatic system.
- May make sense for > ~20 km grid spacing where nonhydrostatic effects are minimal – the model runs more quickly (~20%) in hydrostatic mode.