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# GPS Radio Occultation Data Assimilation

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# Topics covered during this talk

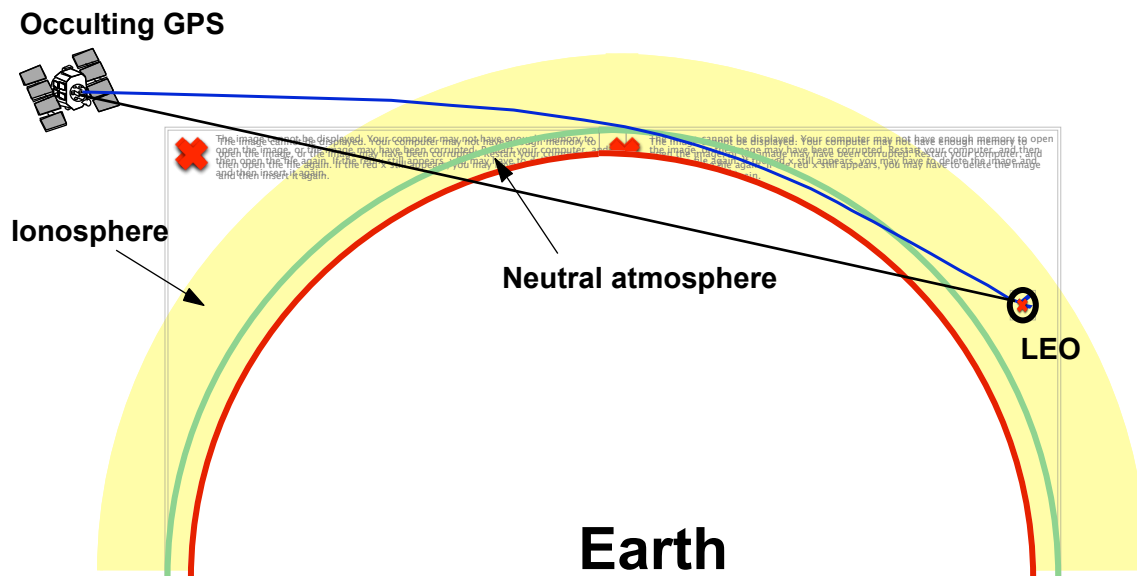
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- Brief introduction to GPS RO technique
- Possible Forward Operators (and what do they mean)
- Assimilation of GPS RO observations within GSI



# Radio Occultation concept

- An occultation occurs when a GPS (GNSS) satellite rises or sets across the limb wrt to a LEO satellite.
- A ray passing through the atmosphere is refracted due to the vertical gradient of refractivity (density).
- During an occultation ( $\sim 3\text{min}$ ) the ray path slices through the atmosphere

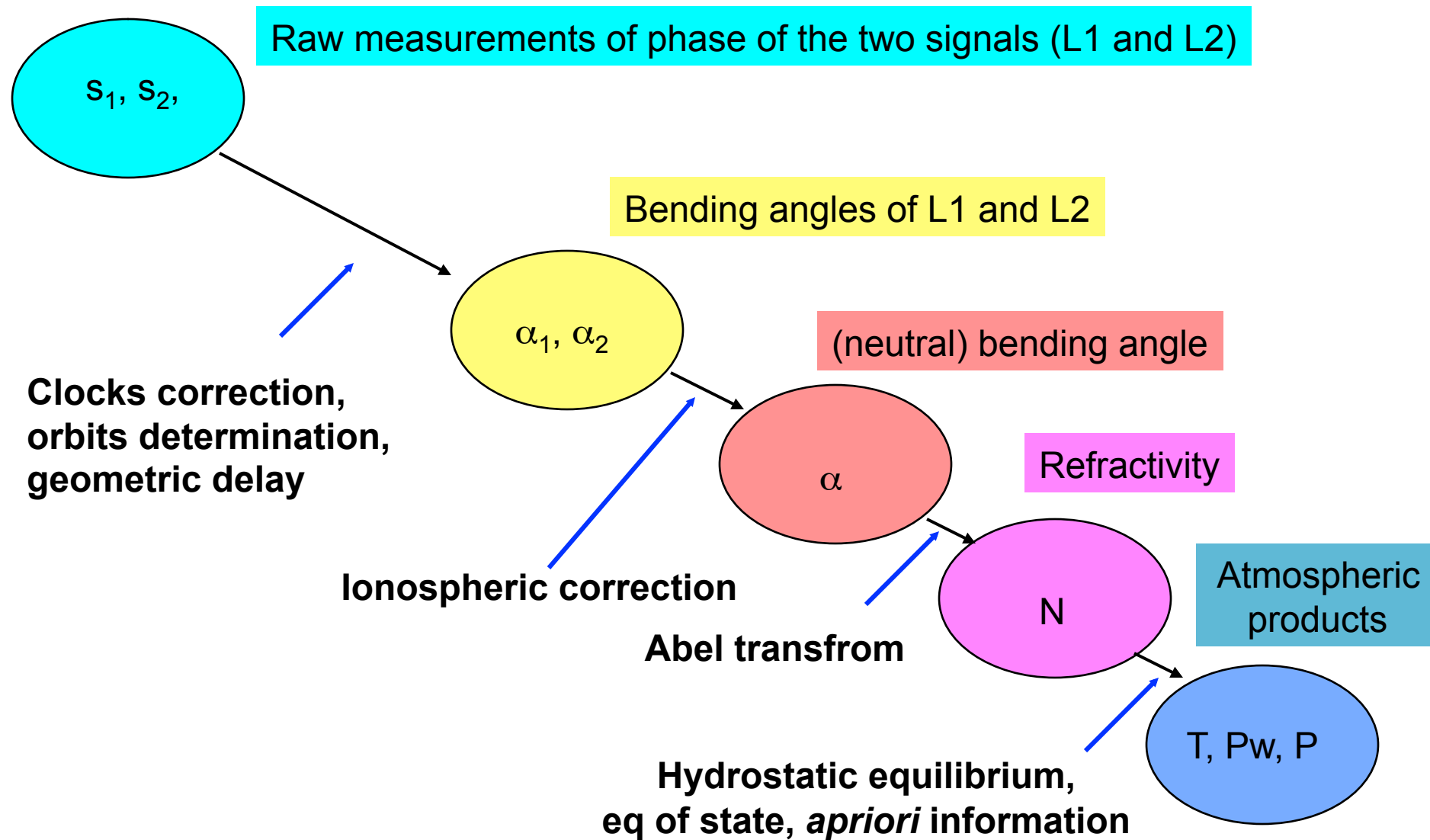


Raw measurement: change of the delay (phase) of the signal path between the GPS and LEO during the occultation. (It includes the effect of the atmosphere).

GPS transmits at two different frequencies:  $\sim 1.6\text{ GHz}$  (L1) and  $\sim 1.3\text{ GHz}$  (L2).



# choice of 'observations'





# Bending angle

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- Correction of the clocks errors and relativistic effects on the phase measurements (time corrections).
  - Compute the Doppler shift (change of phase in time during the occultation).
  - Remove the expected Doppler shift for a straight line signal path to get the atmospheric contribution (ionosphere + neutral atmosphere). [The first-order relativistic contributions to the Doppler cancel out].
  - The atmospheric Doppler shift is related to the known position and velocity of the transmitter and receiver (orbit determination).
  - However, there is an infinite number of atmospheres that would produce the same atmospheric Doppler. (The system is undetermined)
  - Certain assumption needs to be made on the shape of the atmosphere: local spherical symmetry
-



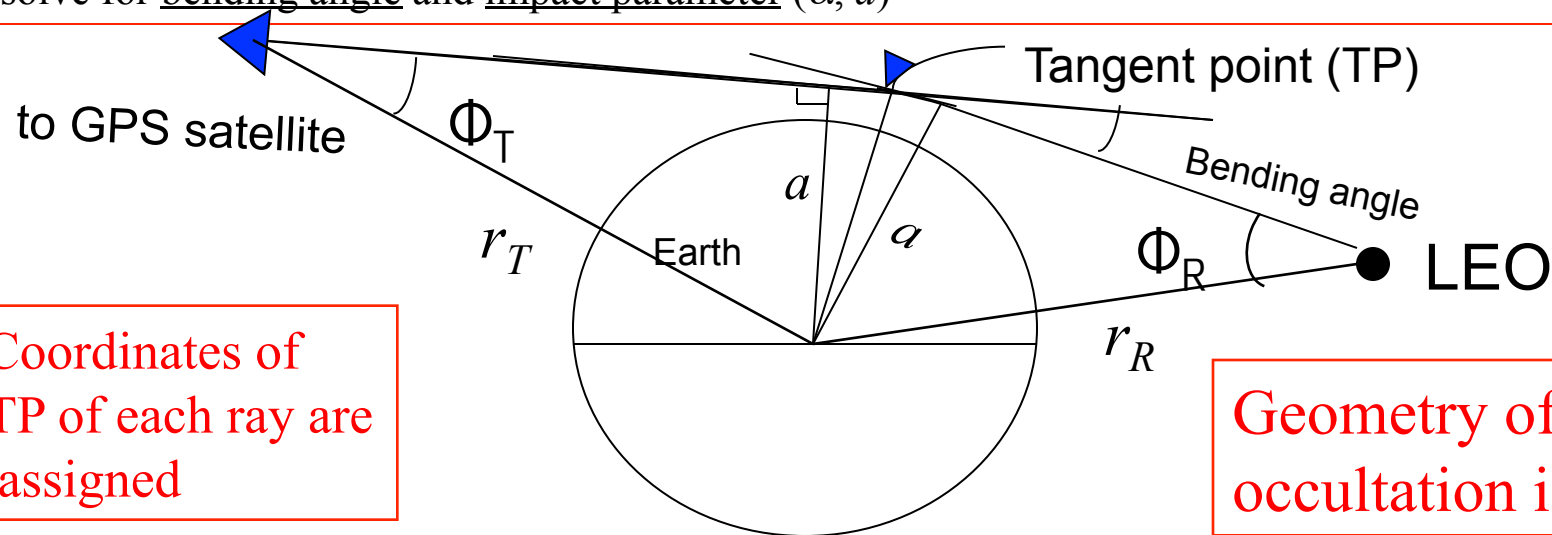
# Bending angle (cont'd)

Spherical symmetry:  $n=n(r) \Rightarrow r n \sin(\Phi) = \text{const} = a$  along the ray path

where  $n$  is the index of refraction ( $c/v$ ),  $r$  is the radial direction,  $\Phi$  is the angle between the ray path and the radial direction and  $a$  is the impact parameter (Bouguer's rule)

At the receiver and transmitter locations  $n_T r_T \sin(\Phi_T) = n_R r_R \sin(\Phi_R) = a$  local symmetry  
(Note that at the tangent point TP,  $n_{TP} r_{TP} = a$ )

With this assumption, the knowledge of the satellite positions & velocities and the local center of curvature (which varies with location on the Earth and orientation of the occultation plane), we solve for bending angle and impact parameter ( $\alpha, a$ )



Coordinates of  
TP of each ray are  
assigned

Geometry of the  
occultation is defined



## (neutral) Bending angle

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- We compute bending angle and impact parameter for each GPS frequency  $(\alpha_1, a_1)$  and  $(\alpha_2, a_2)$ . [The two rays travel slightly different paths because the ionosphere is dispersive].
- For neutral atmospheric retrievals, we compute linear combination of  $\alpha_1$  and  $\alpha_2$  to remove the first-order ionospheric bending ( $\sim 1/f^2$ ) and get the ‘neutral’ bending angle  $\alpha(a)$ 
  - The correction should not be continued above  $\sim 50$ - $90$  km because the signature of the neutral atmosphere might be comparable to the residual ionospheric effects.
  - For ionospheric retrievals, the bending from each frequency is used above  $60$  km.
- Retrieval: **profile of  $\alpha(a)$**  during an occultation ( $\sim 3,000$  rays!)



# Refractivity

- Under (global) spherical symmetry, a profile of  $\alpha(a)$  can be inverted (through an Abel inversion) to recover the index of refraction at the tangent point (ie. we reconstruct the atmospheric refractivity)

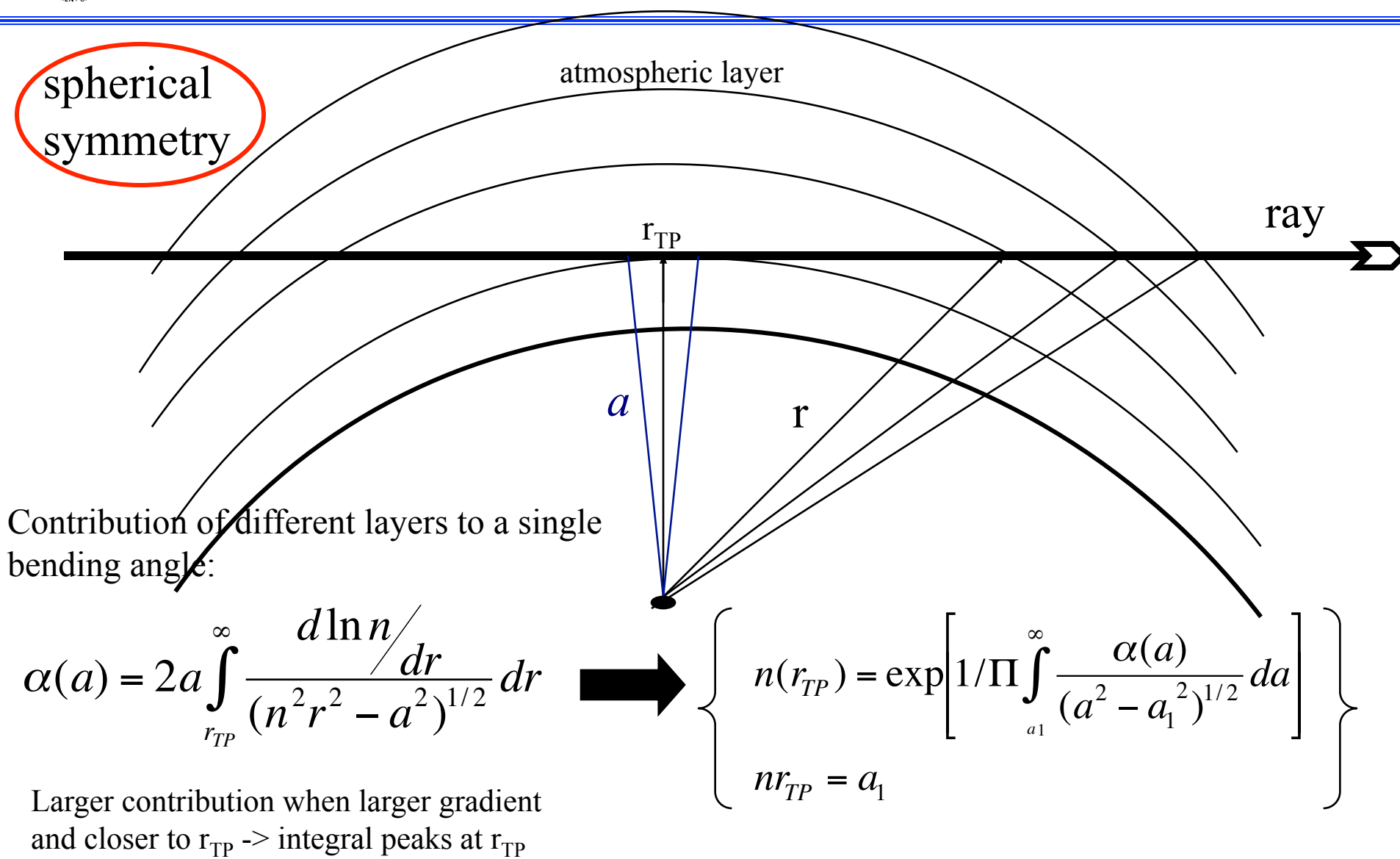
$$n(r_{TP}) = \exp \left[ 1/\Pi \int_{a_1}^{\infty} \frac{\alpha(a)}{(a^2 - a_1^2)^{1/2}} da \right]$$

$$nr_{TP} = a_1$$

- Profile of  $\alpha(a)$  is extrapolated above  $\sim 60$  km (up to  $\sim 150$  km) using climatology information (through statistical optimization) to solve the integral. (The effects of climatology on the retrieved profile are negligible below  $\sim 30$  km).
- Tangent point radius are converted to geometric heights  $z$  (ie. heights above mean-sea level geoid).
- Index of refraction is converted to refractivity:  $N = 10^6 (n - 1)$
- Retrieval: [profile of  \$N\(z\)\$](#)  during an occultation ( $\sim 3,000$  rays)



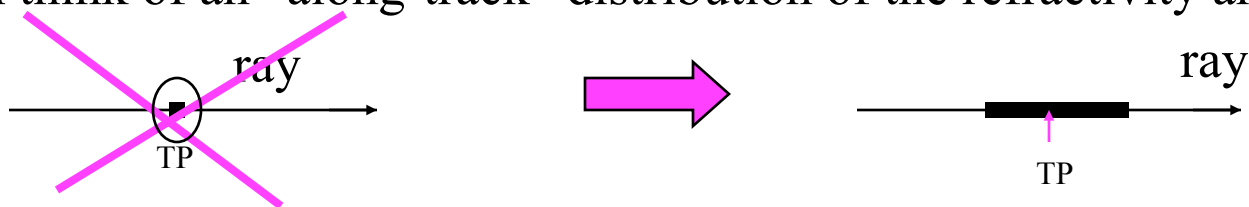
# Rationale for Abel inversion





# Real world....

- If the spherical symmetry assumption was exactly true (ie. no horizontal gradients of refractivity, refractivity only dependent on radial direction)
  - we would not have a job on this business (no weather!)
  - Abel transform would exactly account for and unravel the contributions of the different layers in the atmosphere to a single bending angle.
- However, there is a 3D distribution of refractivity (or 2D) that contributes to a single bending angle and only 1D bending angle (undetermined problem). [Different from the usual nadir-viewing soundings].
- There is contribution from the horizontal gradients of refractivity to a single bending angle. (This can be significant in LT).
- Abel inversion does not account for these contributions along the ray path so there is some residual mapping of non-spherical horizontal structure into the refractivity profile
- We can think of an “along-track” distribution of the refractivity around the TP.





# Atmospheric variables

- At microwave wavelengths (GPS), the dependence of  $N$  on atmospheric variables can be expressed as:

$$N = 77.6 \frac{P}{T} + 3.73 \times 10^5 \frac{P_w}{T^2} - 40.3 \times 10^6 \frac{n_e}{f^2} + O\left(\frac{1}{f^3}\right) + 1.4 \times W_w + 0.6 \times W_i$$

## Hydrostatic balance

$P$  is the total pressure (mb)  
 $T$  is the temperature (K)

## Moisture

$P_w$  is the water vapor pressure (mb)

- important in the troposphere for  $T > 240\text{K}$
- can contribute up to 30% of the total  $N$  in the tropical LT.
- can dominate the bending in the LT.

## Ionosphere

$f$  is the frequency (Hz)  
 $n_e$  electron density ( $\text{m}^{-3}$ )

## Scattering terms

$W_w$  and  $W_i$  are the liquid water and ice content ( $\text{gr}/\text{m}^3$ )

Contributions from liquid water & ice to  $N$  are very small and the scattering terms can be neglected  
**RO technology is almost insensitive to clouds.**



# Atmospheric variables

height of tangent point ↑

**ionosphere**

~ 70 km

**ionospheric term dominates**  
and the rest of the contributions  
can be ignored.  $N$  directly  
corresponds to **electron density**

**“dry” ( $P_w \sim 0$ ) atmosphere**  
 $P$  and  $T$

**neutral  
atmosphere**  
(hydrostatic  
term dominates)

~ 6 km

the ionospheric correction removes the  
1st order ionospheric term ( $1/f^2$ ) because  
GPS has two frequencies.

**“wet” atmosphere ( $P, T, P_w$ )**



# “Dry” atmosphere: $P$ and $T$

- Where the contribution of the water vapor to the refractivity can be neglected ( $T < 240\text{K}$ ) the expression for  $N$  gets reduced to pure density (and  $P = P_d$ ),

$$N(z) = 77.6 \frac{P(z)}{T(z)}$$

- + equation of state:  $\rho(z) = \frac{N(z)m}{77.6R}$  with  $\begin{cases} m = \text{mean molecular mass of dry air} \\ R = \text{gas constant} \end{cases}$

- + hydrostatic equilibrium  $\frac{\partial P}{\partial z} = -g(z)\rho(z)$

- Given a boundary condition (eg.  $P=0$  at 150 km), one can derive
  - Profiles of pressure
  - Profiles of temperature (from pressure and density)
  - Profiles of geopotential heights from the geometric heights (RO provides independent values of pressure and height).



## “Dry” atmosphere: $P$ and $T$ (cont’d)

- When there is no moisture in the atmosphere, the profiles of  $P$  and  $T$  retrieved from  $N$  correspond to the real atmospheric values.

- But when there is moisture in the atmosphere, the expression

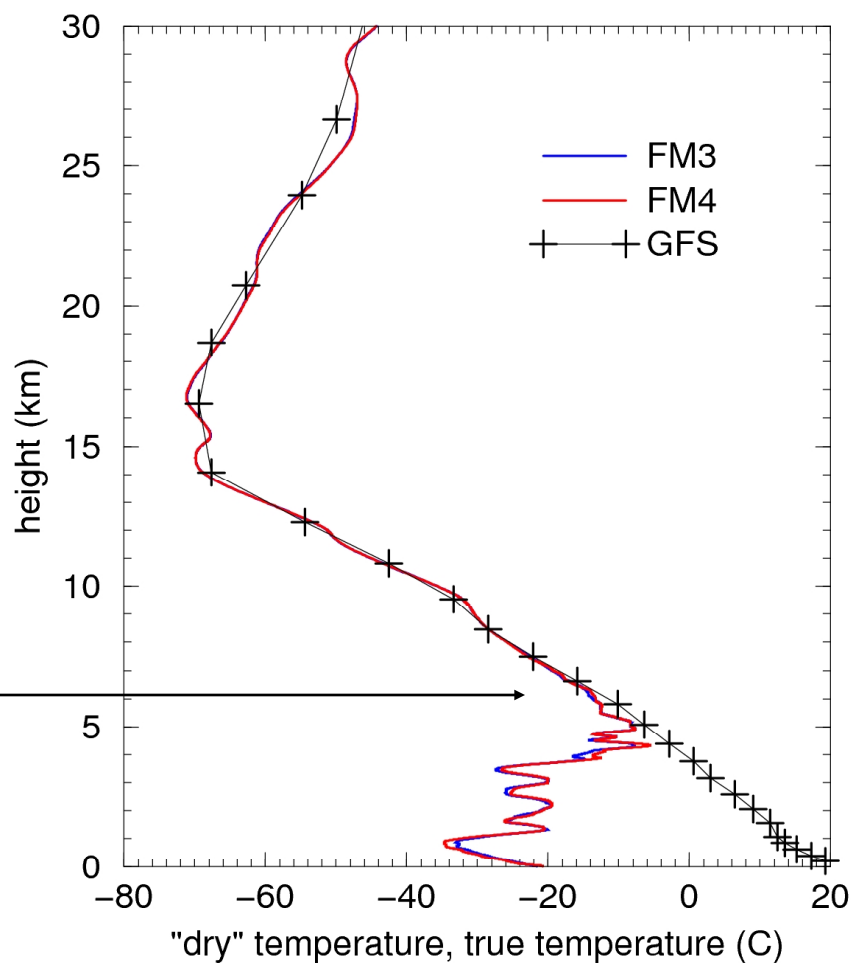
$$N = 77.6 \frac{P}{T}$$

will erroneously map all the  $N$  to  $P$  and  $T$  of a dry atmosphere.

- In other words, all the water vapor in the real atmosphere is replaced by dry molecules that collectively would produce the same amount of  $N$ .
- As a consequence, the retrieved temperature will be lower (cooler) than the real temperature of the atmosphere
- Within the GPS RO community, these profiles are usually referred to “dry temperature” profiles.
- This is confusing and misleading...
- I agree!!!



# Retrieved vs Physical temperature



Moisture  
becomes  
significant



# “Wet” atmosphere: mass and moisture

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- When the moisture contribution to  $N$  is important (middle and lower troposphere), the system is undetermined ( $P, T, P_w$ ).
- We need independent knowledge of temperature, pressure or water vapor pressure to estimate the other two variables.
- Usually, temperature is given by an external source (model) and we solve for pressure and moisture iteratively.
- Alternatively, we can use *apriori* information of pressure, temperature and moisture from a model along with their error characterization (background error covariance matrices) and find the optimal estimates of  $P$ ,  $T$  and  $q$  (variational assimilation)





## Products from RO processing (UCAR/CDAAC)

Profiles of electron density

~ 80-800 km

Profiles of  $\alpha$  ( $a, lat, lon$ ) and  
 $N$  ( $z, lat, lon$ )

~ 0-60 km

“dry” profiles of  $T$  ( $Z, lat, lon$ )  
and  $P$  ( $Z, lat, lon$ )

1dvar “wet” profiles (GFS for real time  
and ECMWF for post-processing) of  
 $T$  ( $Z, lat, lon$ ),  $P$  ( $Z, lat, lon$ ),  $q$  ( $Z, lat, lon$ )

~ 0-40 km

All the products are computed in real-time (for operational weather prediction) and in post-processed mode (more accurate orbits, unified processing software, for climate studies).



# Radio Occultation characteristics

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- Limb sounding geometry complementary to ground and space nadir viewing instruments
    - High vertical resolution ( $\sim 100$  m)
    - Lower 'along-track' resolution ( $\sim 200$  km)
  - All weather-minimally affected by aerosols, clouds or precipitation
  - High accuracy (equivalent to  $\sim 0.5$  Kelvin from  $\sim 7$ -25 km)
  - Equivalent accuracy over ocean than over land
  - No instrument drift, no need for calibration
  - Global coverage
  - No satellite-to-satellite measurement bias
  - Inexpensive compared to other sensors
-



# Assimilation of RO data

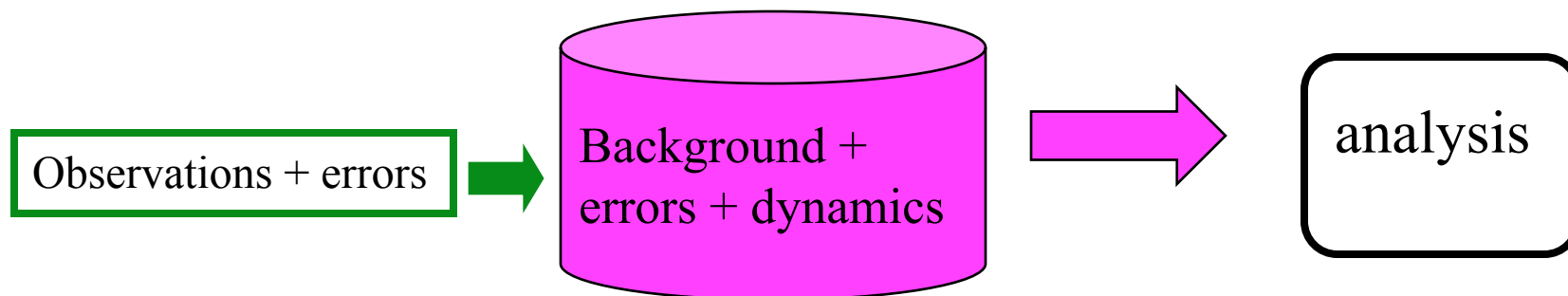
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- The goal is to extract the maximum information content of the RO data, and to use this information to improve analysis of model state variables ( $u$ ,  $v$ ,  $T$ ,  $q$ ,  $P$ , ...etc) and consequent forecasts
  - RO data (bending angles, refractivity, ...) are non-traditional meteorological observations (e.g., wind, temperature, moisture)
  - The ray path limb-sounding characteristics are very different from the traditional meteorological measurements (e.g., radiosonde) or the nadir-viewing passive MW/IR measurements
  - **Basic rule**: the rawer the observation is, the better
-

# Variational assimilation of RO data

- In Variational Analysis (e.g. 3D- or 4D-VAR), we minimize the cost function:

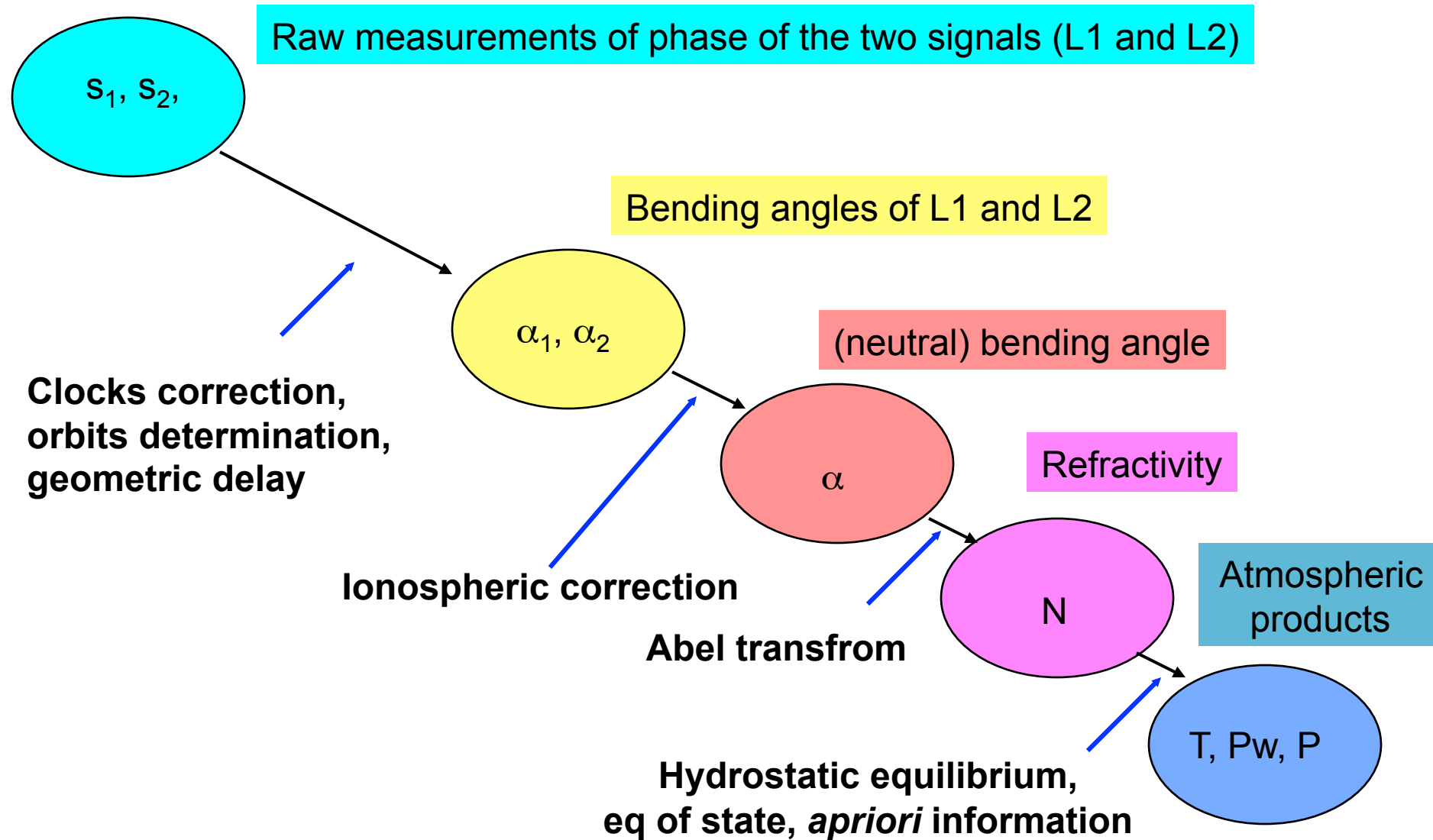
$$J(\mathbf{x}) = (\mathbf{x} - \mathbf{x}_b)^T \mathbf{B}^{-1} (\mathbf{x} - \mathbf{x}_b) + (\mathbf{y}_0 - H(\mathbf{x}))^T (\mathbf{O} + \mathbf{F})^{-1} (\mathbf{y}_0 - H(\mathbf{x})) + J_c$$



- where  $\mathbf{x}$  is the analysis vector,  $\mathbf{x}_b$  is the background vector,  $\mathbf{y}_0$  is the observation vector,  $(\mathbf{O} + \mathbf{F})$  is the observation error covariance matrix ( $\mathbf{F}$  is the representativeness error) and  $\mathbf{B}$  is the background error covariance matrix.
- $H$  is the forward model (observation operator) which transforms the model variables (e.g.  $T$ ,  $u$ ,  $v$ ,  $q$  and  $P$ ) to the observed variable (e.g. radiance, bending angle, refractivity, or other observables).
- We first need to decide what do we want to assimilate

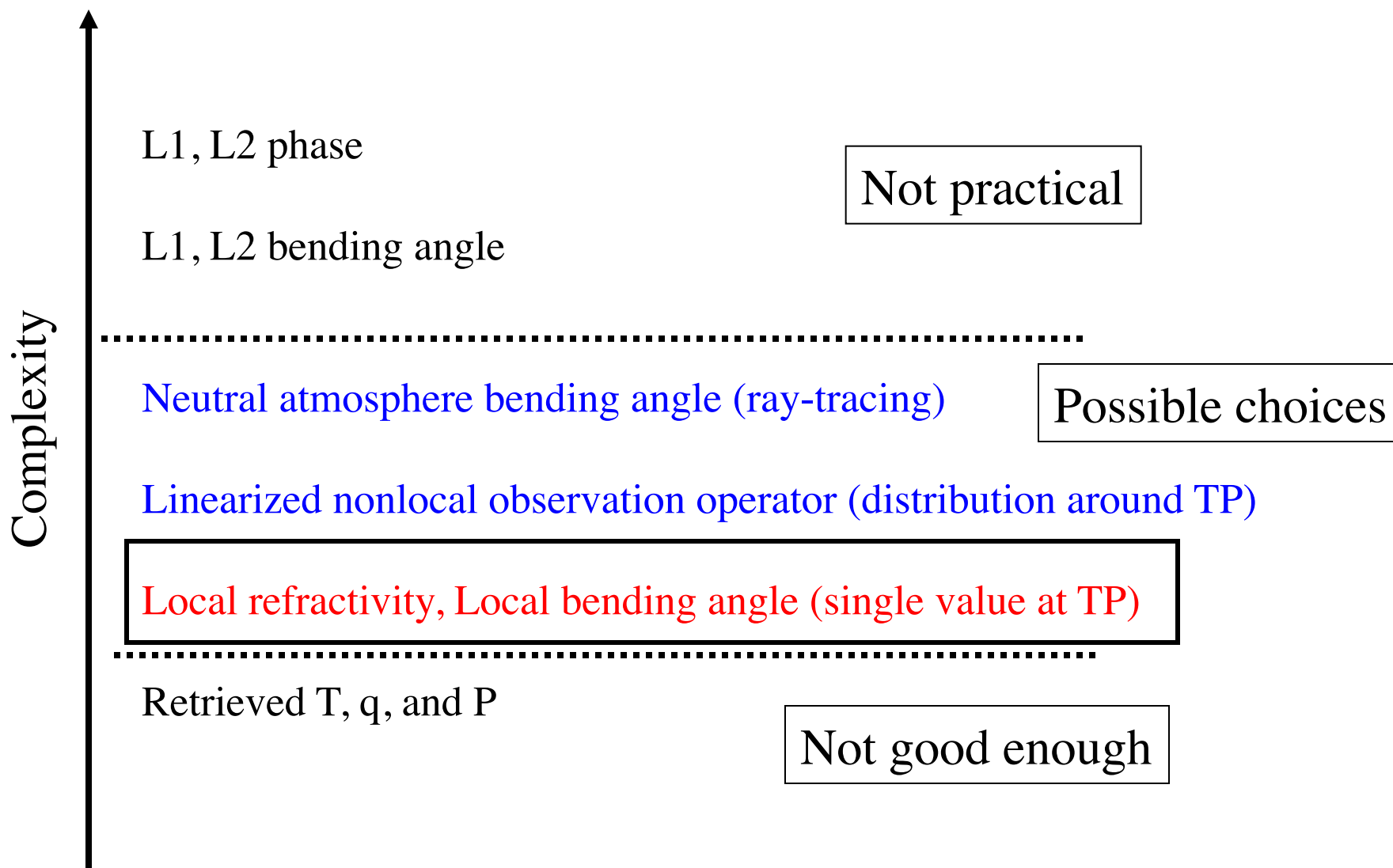


# choice of 'observations'

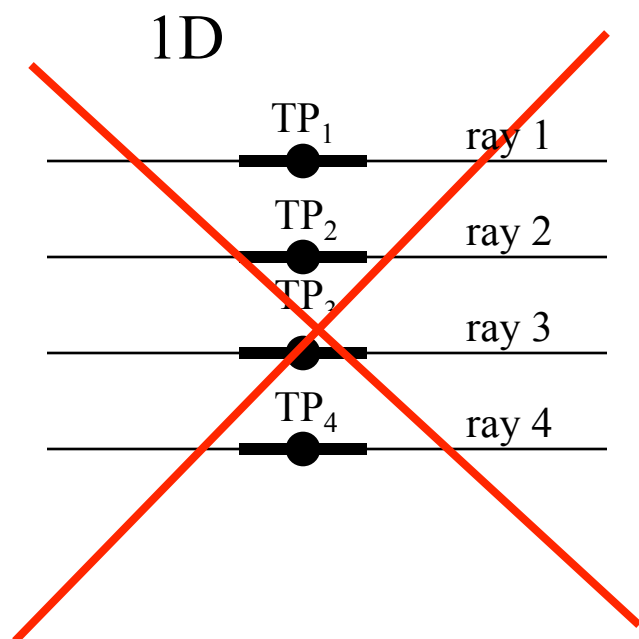




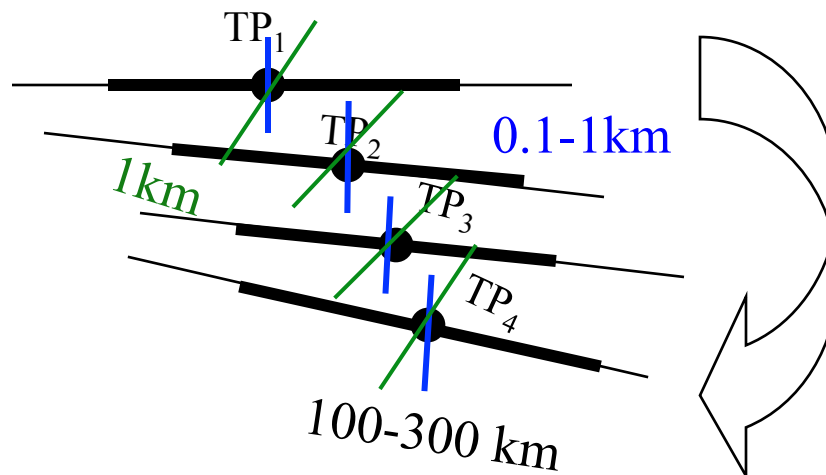
# Choice of observation operators



# Geometry of a GPS RO profile



We need to think in 3D



3,000 rays!!!!

An occultation is not just a vertical profile. The relative motion of the satellites involves an inclination away from the vertical of the surface swept out by the occulting rays (a surface, moreover, that is not in general even a plane)



# Forward Model for refractivity

$$N = 77.60 \frac{P_d}{T} + 70.4 \frac{P_w}{T} + 3.739 \times 10^{-5} \frac{P_w}{T^2}$$

- (1) Geometric height of observation is converted to geopotential height.
- (2) Observation is located between two model levels.
- (3) Model variables of pressure, (virtual) temperature and specific humidity are interpolated to observation location.
- (4) Model refractivity is computed from the interpolated values.
- The assimilation algorithm produces increments of
  - surface pressure
  - water vapor of levels surrounding the observation
  - (virtual) temperature of levels surrounding the observation and all levels below the observation (ie. an observation is allowed to modify its position in the vertical)
- Each observation is treated independently (we account for the drift of the tangent point within a profile)





# Forward Model for bending angle

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$$\alpha(a) = -2a \int_a^{\infty} \frac{d \ln n / dx}{(x^2 - a^2)^{1/2}} dx$$
$$(x = nr)$$

## ■ Make-up of the integral:

- Change of variable to avoid the singularity

$$x = \sqrt{a^2 + s^2}$$

- Choose an equally spaced grid to evaluate the integral by applying the trapezoid rule



## Forward Model for bending angle (cont'd)

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- Compute model geopotential heights and refractivities at the location of the observation
- Convert geopotential heights to geometric heights
- Add radius of curvature to the geometric heights to get the radius:  $r$
- Convert refractivity to index of refraction:  $n$
- Get refractional radius ( $x=nr$ ) and  $d\ln(n)/dx$  at model levels and evaluate them in the new grid. We make use of the smoothed Lagrange-polynomial interpolators to assure the continuity of the FM wrt perturbations in model variables.
- Evaluate the integral in the new grid.
- Each observation is treated independently (we account for the drift of the tangent point within a profile)



# GPSRO code structure within GSI

Select type of observation & satellites

regression\_namelists.sh

rungsi62\_prod.sh

Global\_convinfo.txt

**Read obs**

read\_gps.f90

**Forward**

setupref.f90 & setupbend.f90  
genstats.f90

**TL & adjoint**

intgps.f90  
stpgps.f90



# Submitting a job

Select type of ‘observation’ in `regression_namelists.sh`

```
&OBS_INPUT
```

```
.....  
dfile(10)='gpsrobufr', dtype(10)='gps_ref', dplat(10)=' ', dsis(10)='gps_ref', dval(10)=1.0, dthin(10)=0,  
dsfcalc(10)=0,
```

```
.....  
$OBSINPUT
```

```
/
```

**\*\*Use ‘gps\_bnd’ for bending angle (this capability is currently being upgraded, do not use it yet!!)**



# global\_convinfo.txt

Select what missions to assimilate (iuse=1) or monitor (iuse=-1)

!otype type sub iuse twindow numgrp ngroup nmitter gross ermax ermin var\_b var\_pg ithin rmesh pmesh npred

gps_ref	401	0	-1	3.0	0	0	0	10.0	10.0	1.0	10.0	0.000000	0	0.	0.	0
gps_ref	004	0	1	3.0	0	0	0	10.0	10.0	1.0	10.0	0.000000	0	0.	0.	0
gps_ref	041	0	-1	3.0	0	0	0	10.0	10.0	1.0	10.0	0.000000	0	0.	0.	0
gps_ref	722	0	1	3.0	0	0	0	10.0	10.0	1.0	10.0	0.000000	0	0.	0.	0
gps_ref	723	0	-1	3.0	0	0	0	10.0	10.0	1.0	10.0	0.000000	0	0.	0.	0
gps_ref	740	0	1	3.0	0	0	0	10.0	10.0	1.0	10.0	0.000000	0	0.	0.	0
gps_ref	741	0	1	3.0	0	0	0	10.0	10.0	1.0	10.0	0.000000	0	0.	0.	0
gps_ref	742	0	1	3.0	0	0	0	10.0	10.0	1.0	10.0	0.000000	0	0.	0.	0
gps_ref	743	0	1	3.0	0	0	0	10.0	10.0	1.0	10.0	0.000000	0	0.	0.	0
gps_ref	744	0	1	3.0	0	0	0	10.0	10.0	1.0	10.0	0.000000	0	0.	0.	0
gps_ref	745	0	1	3.0	0	0	0	10.0	10.0	1.0	10.0	0.000000	0	0.	0.	0
gps_ref	820	0	-1	3.0	0	0	0	10.0	10.0	1.0	10.0	0.000000	0	0.	0.	0
gps_ref	042	0	-1	3.0	0	0	0	10.0	10.0	1.0	10.0	0.000000	0	0.	0.	0

COSMIC

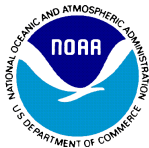
\*\*Select options for 'gps\_bnd' to assimilate bending angle



## read\_obs.f90

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- Reads refractivity/bending angle observations from bufr files (~ 300 values are available in each profile of refractivity and bending angle)
- Applies initial quality control (as specified by provider)
- Performs ‘sanity’ QC (correct date, acceptable values for variables, etc)
- Assigns initial error to the observations
- Stores information and prints out basic output (# profiles/mission, total number of good/bad observations, refractivity/bending angle selection, etc)



# setuprhsall.f90

```
! Set up GPS local refractivity data
else if(ditype(is) == 'gps')then
  if(obstype=='gps_ref')then
    call setupref(lunin,mype,awork(1,i_gps),nele,nobs,toss_gps_sub,is,init_pass,last_pass)

! Set up GPS local bending angle data
else if(obstype=='gps_bnd')then
  call setupbend(lunin,mype,awork(1,i_gps),nele,nobs,toss_gps_sub,is,init_pass,last_pass)
end if
end if
```

```
! Finalize qc and accumulate statistics for GPSRO data
call genstats_gps(bwork,awork(1,i_gps),toss_gps_sub,conv_diagsave,mype)
```



# Forward operator for $N(z)$

## ■ Computed in setupref.f90

- Pressure at the location of the observations
- Simulate observed refractivity (local, 3-term expression, Bevis coefficients)
- Tune for representativeness errors
- Apply quality control procedures
- Observations above 30 km are rejected
- Calculate and store the Jacobians for minimization
- Store diagnostic information

! Compute guess local refractivity

```
fact=(one+fv*qrefges)
```

```
pw=eps+qrefges*(one-eps)
```

```
k4=n_c-n_a
```

```
nrefges1=n_a*(pressure(i)/trefges)*fact
```

```
nrefges2=n_b*qrefges*pressure(i)*fact**2/(trefges**2*pw)
```

```
nrefges3=k4*fact*qrefges*pressure(i)/(trefges*pw)
```

```
nrefges=nrefges1+nrefges2+nrefges3 !total refractivity
```



# Forward operator for $\alpha(a)$



## ■ Computed in setupbend.f90

- Refractivity index-radius product
- Simulate observed bending angle (calls other subroutines to compute Lagrangians interpolators)
- Tune for representativeness errors
- Apply quality control procedures
- Expanding maximum height for rejection of an observation is under current testing
- Calculate and store the Jacobians for minimization
- Store diagnostic information

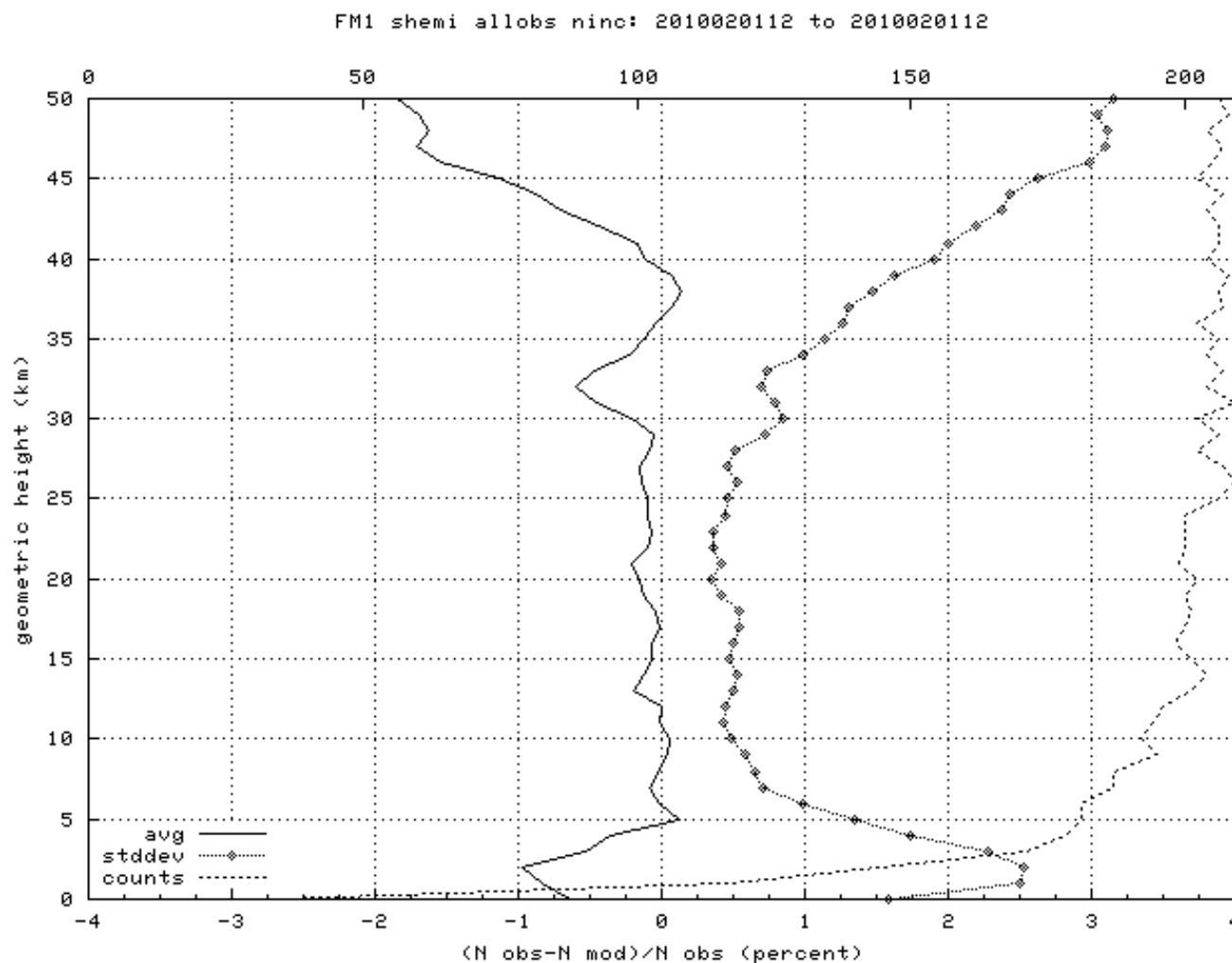


## genstats.f90

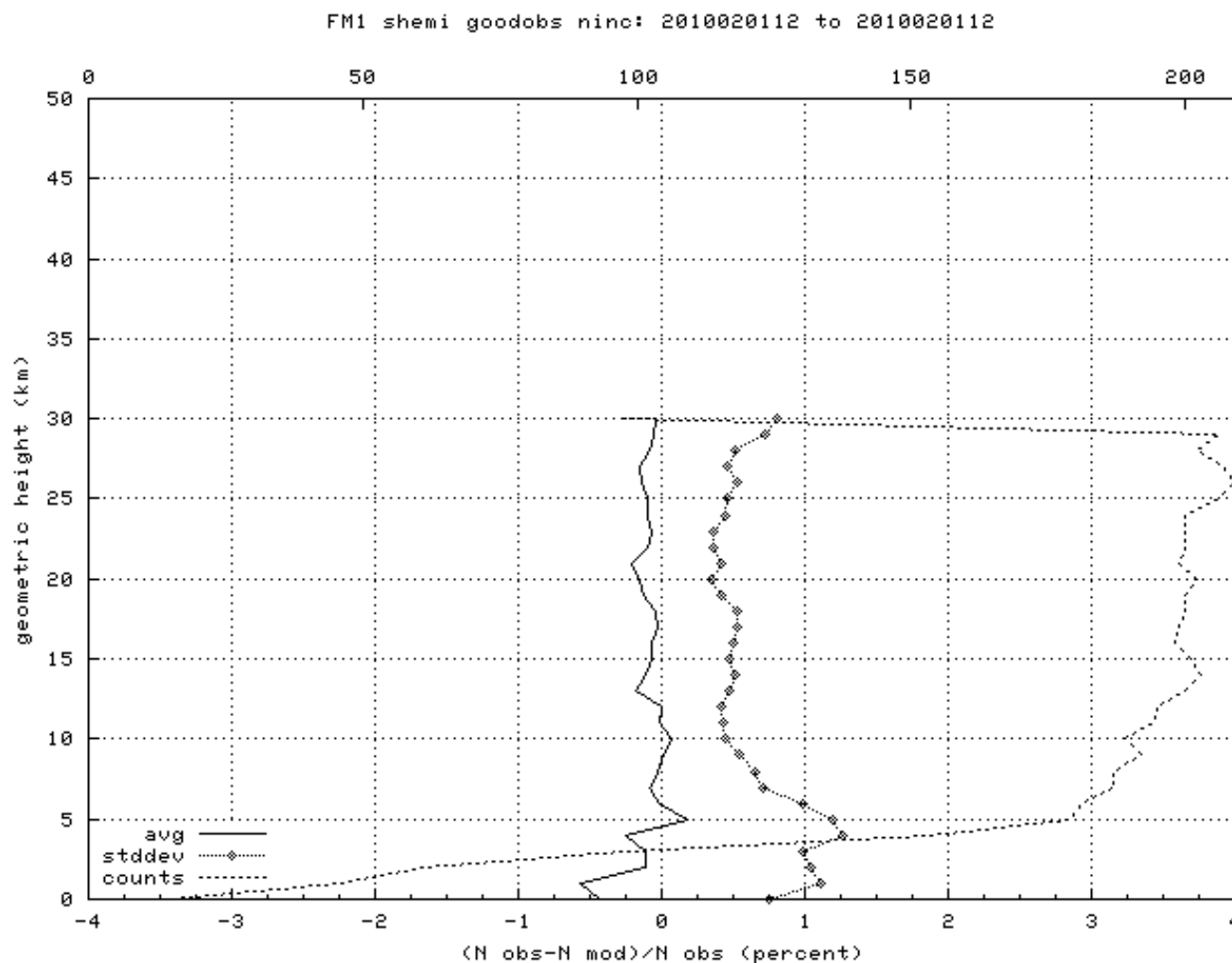
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- Unifies QC procedures among tasks – it might modify the QC of the observations
- Adjusts observation error ratio based on “superobs” factor
- Accumulates statistics
- If requested, writes information to diagnostic file  
(diag\_conv\_ges.2010020112.Z, diag\_conv\_anl.2010020112.Z)

# Refractivity example (before QC)

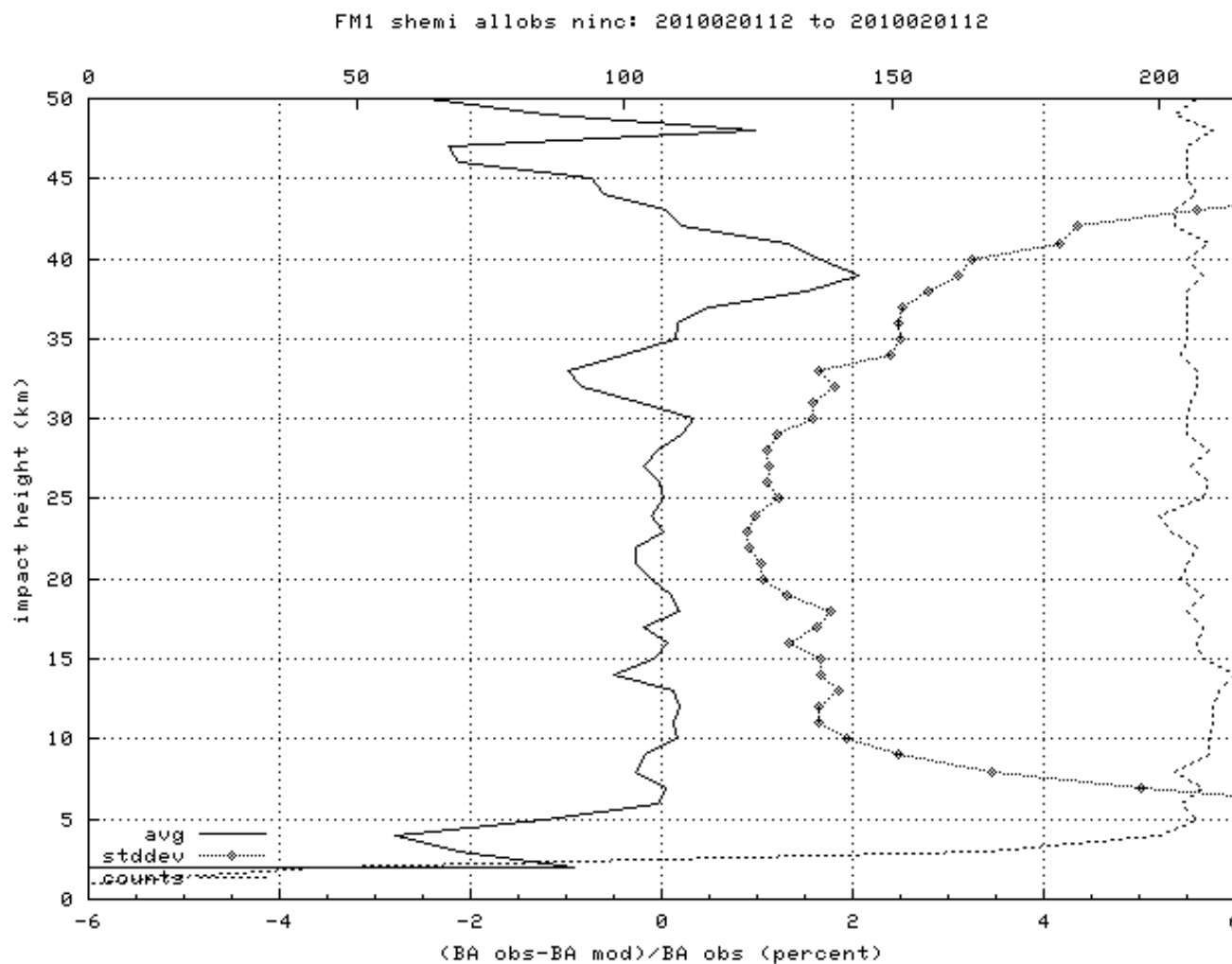


# Refractivity example (after QC)





# Bending angle example (before QC)

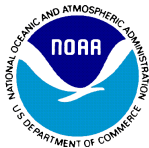




## intgps.f90 & stpgps.f90

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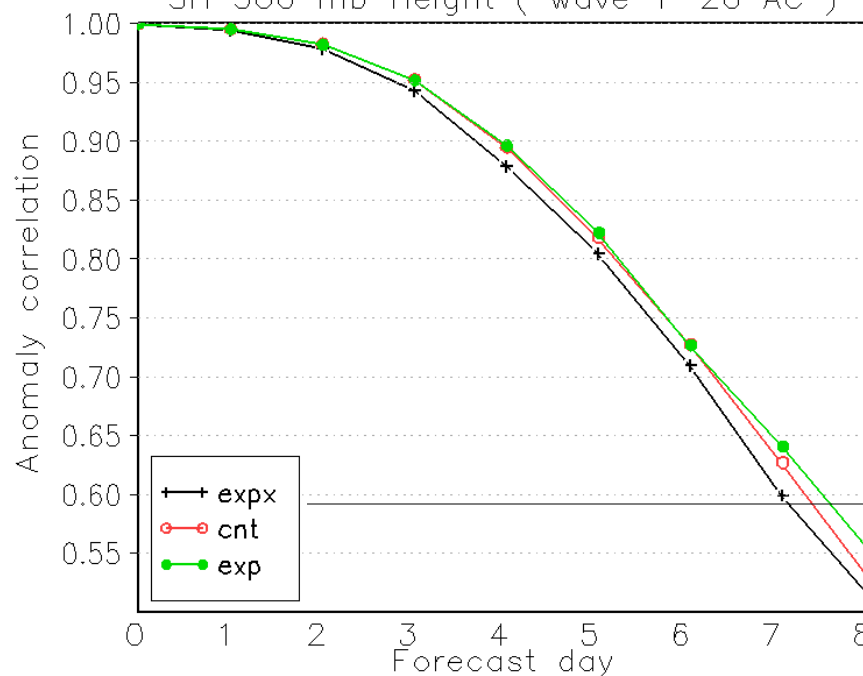
- Drive the algorithms for GPS RO component during the minimization
- Nothing special, similar to the the minimization routines for other observations
- Diagnostic information for GPS RO observations is stored in unit 212



# Recent impact with COSMIC

- AC scores (the higher the better) as a function of the forecast day for the 500 mb gph in Southern Hemisphere
- 40-day experiments:
  - **expx** (NO COSMIC)
  - **cnt** (old RO assimilation code - with COSMIC)
  - **exp** (ops, updated RO assimilation code - with COSMIC)

AVERAGE FOR 00Z25MAR2008 – 00Z30APR2008  
SH 500 mb Height ( wave 1–20 AC )



**COSMIC provides 8 hours of gain in model forecast skill starting at day 4 !!!**



# References of GPSRO in GSI

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- **Cucurull, L.** and J. C. Derber, 2008: Operational implementation of COSMIC observations into the NCEP's Global Data Assimilation System, *Wea. Forecasting*, **23**, 4, 702-711.
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- Anthes, R. A., P.A. Bernhardt, Y. Chen, **L. Cucurull**, K. F. Dymond, D. Ector, S. B. Healy, S.-P. Ho, D. C. Hunt, Y.-H. Kuo, H. Liu, K. Manning, C. McCormick, T. K. Meehan, W. J. Randel, C. Rocken, W. S. Schreiner, S. V. Sokolovskiy, S. Syndergaard, D. C. Thompson, K. E. Trenberth, T.-K. Wee, N. L. Yen, and Z. Zeng, 2008: The COSMIC/FORMOSAT-3 Mission: Early Results, *BAMS*, **89**, 313-333.
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